Two's Complement Math (E)

Binary Addition and Subtraction Review

The topics of binary addition and subtraction are normally well covered in basic digital classes. As such, it is only presented here as an intro to two's complement addition and subtraction.

Addition Rules Review

			1
		1	1
0	0	1	1
+ 0	+ 1	+ 1	+ 1
0	1	1 0	1 1
$0 + 0 = 0$	$0 + 1 = 1$	$1 + 1 = 10$	$1 + 1 + 1 = 11$
		(0 carry 1)	(1 carry 1)

Note in the following review examples that the answers were verified in several ways. All good engineers constantly verify their work!

Binary Addition Example 1

Carry ⇒		1	1		1	1		
Augend ⇒				1	0	1	1_2	(11_{10})
Addend ⇒	+		1	1	0	1	1_2	(27_{10})
Sum ⇒		1	0	0	1	1	0_2	(38_{10})

check ⇒ $2^5(32)$ $2^2(4)$ $2^1(2)$ = (38_{10})

Binary Addition Example 2

Carry ⇒		1				1	1	1		
Augend ⇒			1	1	0	0	1	1	1_2	(103_{10})
Addend ⇒	+		1	0	0	0	1	1	1_2	(71_{10})
Sum ⇒		1	0	1	0	1	1	1	0_2	(174_{10})
check ⇒		$2^7(128)$		$2^5(32)$		$2^3(8)$	$2^2(4)$	$2^1(2)$	=	(174_{10})

Note that there **isn't a defined word length for the binary numbers in this kind of addition**. If the addition causes an overflow, then the word length just gets larger. In addition, there isn't a requirement that the two binary numbers must be the same length. Of course, if you were to do this with a computer algorithm, you would be limited by the word length of the computer system.

Binary Subtraction Rules Review

$0 - 0$	$1 - 0$	$1 - 1$	$\overset{1}{\cancel{0}} - 1$
0	1	0	1
$0 - 0 = 0$	$1 - 0 = 1$	$1 - 1 = 0$	$0 - 1 = -1$ **2 borrowed from next column to make 10_2** (*NEGATIVE*)

Binary Subtraction Example 1

	0	1					
minuend	~~1~~	[1]~~0~~	[1]0	1	1	1_2	(39_{10})
Subtrahend		1	1	1	0	1_2	(29_{10})
Difference		0	1	0	1	0_2	(10_{10})

***Verified* by Observation**

Binary Subtraction Example 2

					0	[1]0					
Minuend	$\Rightarrow$	1	0	1	~~1~~	~~1~~	[1]0	1	1	1_2	(375_{10})
Subtrahend	$\Rightarrow$			1	0	1	1	1	0	0_2	(92_{10})
Difference	$\Rightarrow$	1	0	0	0	1	1	0	1	1_2	(283_{10})
Check	$\Rightarrow$	256				16	8		2	1	$= 283_{10}$

2's Complement Math

Now that we can find the negative of a number, we can perform the addition by adding a negative number to a positive number when we want to subtract **instead of adding in additional hardware to subtract.**

2's Complement Math Examples:

Let's start out by determining the positive and negative two's complements of **23** and **30** for <u>N = 6</u>. We will use these numbers in different combinations to explore **2's Complement math.**

$$N = 6$$

$$\begin{array}{lll|lll} +23 & \Rightarrow & 0,10111_{2cm} & 0,11110_{2cm} & \Leftarrow & +30 \\ \hline -23 & \Rightarrow & 1,01001_{2cm} & 1,00010_{2cm} & \Leftarrow & -30 \end{array}$$

<u>The first thing to remember in any kind of Complement math is that now **ALL NUMBERS which are to be added or subtracted MUST BE THE SAME (N) VALUE**</u>. **All carry's above the N value will be discarded.**

2's Complement Math Example 1

Problem: Perform the operation -30 + 23 using 2's complement math. N = 6

carry ⇒				1	1		
(−30)		1,	0	0	0	1	0_{2cm}
+23	+	0,	1	0	1	1	1_{2cm}
(−7)		1,	1	1	0	0	1_{2cm}
		-2^5	2^4	2^3			2^0
−7	⇔	−32	+16	+8			+1

2's Complement Math Example 2

Problem: Perform the operation +30 + (-23) using 2's complement math. $N = 6$

carry ⇒	~~1~~	1	1				
30		0,	1	1	1	1	0_{2cm}
+(−23)		1,	0	1	0	0	1_{2cm}
+ 7		0,	0	0	1	1	1_{2cm}
	⇑						
	discard						

Note that the overflow was discarded since it was <u>greater than N=6</u>. "In other words, it went into the proverbial <u>BIT BUCKET</u>!"

Let's look at another example with **N=6**.

2's Complement Math Example 3

Problem: Perform the operation (-30) + (-23) using 2's complement math. N = 6

carry	⇒	~~1~~							
(-30)			1,	0	0	0	1	0_{2cm}	
+ (-23)			1,	0	1	0	0	1_{2cm}	**DOES NOT VALIDATE!!!!**
(-53)	≠		0,	0	1	0	1	1_{2cm}	
???		⇑ discard							

QUESTION: **Something is wrong with this example. The answer should be -53_{10} but the result is +11_{10}. What happened?**

ANSWER: **It should be obvious! You can't represent a 53 with 5 bits and a sign bit! +53 = $0{,}110101_{2cm}$. That's 7 bits including the sign bit!**

This has been an object lesson on what happens if the N value is too low!!!

So, in order to perform this addition, we have to change the value of N. Let's change the **N** value to (**N = 8**). When we do this, all of the two's complement numbers must be re-computed.

We have to recalculate our numbers for the new N = 8 criteria!

+30	⇒	$0{,}0011110_{2cm}$	and	$0{,}0010111_{2cm}$	⇐	+23
–30	⇒	$1{,}1100010_{2cm}$	and	$1{,}1101001_{2cm}$	⇐	- 23

2's Complement Math Example 4

Problem: Perform the operation (-30)+(-23) using 2's complement math. N = 8

carry	⇒	~~1~~	1	1						
(-30)			1,	1	1	0	0	0	1	0_{2cm}
+ (-23)			1,	1	1	0	1	0	0	1_{2cm}
(-53)			1,	1	0	0	1	0	1	1_{2cm}
-53	=	⇑ discard	-128	+64			8		2	1

A final 2's Complement Math Example:

2's Complement Math Example 5

Problem: Perform the operation 66_{10} - 24_{10} using 2's complement math. N = 8

carry ⇒	~~1~~	1								
66		0,	1	0	0	0	0	1	0_{2cm}
+ -24		1,	1	1	0	1	0	0	0_{2cm}
42		0,	0	1	0	1	0	1	0_{2cm}
42 =	⇑ discard			+32		8		2	