8 and 16's Complement Math (F)

Octal Addition and Subtraction Introduction

The key to addition and subtraction in **Octal** and **Hex** is to remember that when you carry or borrow in **decimal math**, you are **carrying or borrowing** the radix, **ten**. Likewise, in

- **Octal**
  - you are **carrying or borrowing** the radix **eight** and in
- **Hex**
  - **you are carrying or borrowing** the radix **16**.

There are many ways to **represent a borrow or a carry**. Quite often I represent a borrow in **base 10**, making sure that **<u>I write the base number plainly to remember that is what I'm doing</u>**. This is because I find it a lot easier to work in base 10. **Why do things the hard way?** Let's look at a few examples!

<u>Octal Addition Example</u>

$$\begin{array}{cccccccccc} & 1 & & & & & 1 & 1 & 1 & \\ & 4 & 7 & 7_{10} & \Leftrightarrow & & & 7 & 3 & 5_8 \\ + & & 5 & 1_{10} & \Leftrightarrow & + & & & 6 & 3_8 \\ \hline & 5 & 2 & 8_{10} & \Leftrightarrow & & 1 & 0 & 2 & 0_8 \end{array}$$

$$1020_8 = (1 * 8^3) + (2 * 8^1) = 528_{10}$$

1st column: ➔ $5 + 3 = 8_{10} = 10_8$ so the **0** would be recorded and the **1** would be carried.

2nd column: ➔ $3 + 6 + 1 = 10_{10} = 12_8$ so the **2** is recorded and the **1** is carried to the next column.

3rd column: ➔ $1 + 7 = 8_{10} = 10_8$ so the **0** gets recorded and the **1** gets carried.

<u>In standard math without a required N value, the carry isn't discarded.</u>

Octal subtraction example:

$$
\begin{array}{cccc}
 & & 7 & 13 \\
\hline
 & 2 & 8 & 3_{10} \\
- & & 2 & 9_{10} \\
\hline
 & 2 & 5 & 4_{10}
\end{array}
\qquad\Bigg|\qquad
\begin{array}{cccc}
 & 3 & 10_{10} & \\
 & & \cancel{2} & 11_{10} \\
\hline
 & \cancel{4} & \cancel{3} & \cancel{3}_8 \\
- & & 3 & 5_8 \\
\hline
 & 3 & 7 & 6_8
\end{array}
$$

$$\underbrace{(3 * 8^2)}_{(3*64)} + \underbrace{(7 * 8)}_{56} + \underbrace{(6 * 1)}_{6}$$

$$192 \quad + \quad 62 \quad = 254_{10}$$

1st column: Since you can't subtract **5** from **3**, you need to borrow the **radix (8)** from the next column. Then **3 + 8 = 11**$_{10}$ and then **11 - 5 = 6**.

2nd column: Since you can't subtract **3** from **2**, you need to borrow the **radix (8)** from the next column. Now, **2 + 8 = 10**$_{10}$ and **10 - 3 = 7**.

Eight's Complement System

Just as with two's complement, we can also convert the Octal numbers in our math problem into the **Eight's Complement System**. Then we can save ourselves hardware costs by first finding the negative of the subtrahend and then adding the two together. The rules for the **8's Complement System** are as follows:

- Starting with an **Octal** number, determine the number of digits you need and pad with zero's. If you are going to perform math, **<u>all the numbers must have the same number of digits</u>**. Once the number is at the correct **N – 1** value, add in the **Octal positive sign bit, 0,** followed by a comma. The existence of a sign bit followed by a comma and the subscript 8 attached to the number together identify the entire number as being in the **8's Complement System**.

- You now have a **positive 8's complement number**.

If it is desired to find the **negative** of this positive 8's complement number, then:

- **To find the negative of an 8's complement number**, start with the **LSB**. Copy all **0**'s until you get to the 1st non-zero number.
- Determine what number would need to be added to this **1st non-zero number** which would result in an **8**. This is the number you bring down. For instance, if the number were a **3**, you would have to add a **5** to get **8** so you would bring down a **5**.
- Repeat for the rest of the numbers except that for the rest, **you write down the number which would result in a 7 when added**, not an 8. For example, if one of the next numbers was a **2**, you would need to **add 5 to 2 to get 7**, so you would bring down a **5**.
- The resulting number is the **negative 8's complement** number equivalent of the number you started with. It will have a **negative sign bit, (7,).**

Eights Complement math example:

Rework the previous example, **433_8-35_8** in the **8's Complement System**:

- First choose an **N value** if it is not already known. We will use **N = 4** for this subtraction.
- Next, pad the **433** with a positive sign digit **(0),**
- Pad the **35** with an extra **(0)** to cause number to meet the **N – 1 =3** requirment.
- Place an **8's Complement positive sign bit, 0,** followed by a **comma,** on the left side of the **035.**
- The two numbers now have the same number of digits **(N).**

Note
We will assume that if the number leads with a 0 or a 7, is followed by a ',', and ends with a subscript of 8, that it is in the 8cm system.

- Next, since this is a subtraction, it is necessary to find the negative of the subtrahend, the **$0{,}035_8$. That negation is shown here:**

$$\begin{array}{cccc} 0, & 0 & 3 & 5_8 \\ \hline 7, & 7 & 4 & 3_8 \end{array}$$

  - Note in the negation above, the right most column totals **8** and each of the other columns total **7.**

- Finally we can perform the subtraction by adding the two numbers.

$$\begin{array}{ccccc} \cancel{1} & 1 & & & \\ \hline & 0, & 4 & 3 & 3_8 \\ + & 7, & 7 & 4 & 3_8 \\ \hline & 0, & 3 & 7 & 6_8 \end{array}$$

Note **that 4 + 7 = 11_{10}.**
8 goes into 11 one time with
a remainder of 3 which gives = 13_8.
Also **note that the final carry,**
8_{10} = 10_8 was discarded.

Base 16 (HEX) Math

We can repeat the same processes that we used with the **OCTAL** system in **HEX Math**; we just have to remember that **we are dealing with a radix of 16, vice 8.**

HEX Addition Example

Add **0x5BA** and **0xC7** in the Hex system.

$$\begin{array}{rrrrl} & 1 & 1 & & \\ 1 & 4 & 6 & 6_{10} & \Leftrightarrow \\ + & 1 & 9 & 9_{10} & \Leftrightarrow \\ \hline 1 & 6 & 6 & 5_{10} & \Leftrightarrow \end{array} \quad \begin{array}{rrrr} & 1 & 1 & \\ & 5 & B & A_{16} \\ + & & C & 7_{16} \\ \hline & 6 & 8 & 1_{16} \end{array}$$

$$\left(6 \bullet 16^2\right) + \left(8 \bullet 16^1\right) + \left(1 \bullet 16^0\right)$$

$$\left(6 \bullet 256\right) + \left(8 \bullet 16\right) + \left(1 \bullet 1\right)$$

$$1,536 + 128 + 1 = 1,665_{10} \Leftarrow \textit{Verified}$$

- **1ST column: →** **A + 7 = 10 + 7 = 17_{10}.**
  - **16** will go into **17 one time** with **1 remainder** so **17_{10} = 11_h.**
  - Therefore, you bring down the **1 and carry the 1.**
- **2nd column: →** **B + C = 11 + 12 + 1 = 24_{10}**
  - **16** will go into **24 one time** with **8 remainder** so **24_{10} = 18_h**
  - Therefore, you bring down the **8 and carry the 1.**

HEX Subtraction Example

Subtract **0xDA** from **0xBC7**:

$$
\begin{array}{rcccc|cccc}
 & & & & & & A & 27_{10} & \\
 & 2 & 9 & {}^{1}0 & & & & \cancel{B} & 23_{10} \\
\hline
 & \cancel{3} & {}^{1}\cancel{0} & \cancel{1} & {}^{1}5_{10} & & \cancel{B} & \cancel{C} & \cancel{7}_{h} \\
- & & 2 & 1 & 8_{10} & - & & D & A_{h} \\
\hline
 & 2 & 7 & 9 & 7_{10} & & A & E & D_{h}
\end{array}
$$

$$\left(A \bullet 16^{2}\right) + \left(E \bullet 16^{1}\right) + \left(D \bullet 16^{0}\right)$$

$$\left(10 \bullet 256\right) + \left(14 \bullet 16\right) + \left(13 \bullet 1\right)$$

$$2560 + 224 + 13 = 2797_{10} \Leftarrow \textit{Verified}$$

- **1ST column:** ➔ You can't subtract **A(10)** from **7**,
  - so you have to **borrow the radix (16)** from the next column turning the **C** into a **B**.
  - Then **16 + 7 = 23_{10}** .
  - Now, **23 - A = 23 - 10 = 13 (D_h)**.
- **2nd column:** ➔ You can't subtract **D(13)** from **B(11)**,
  - so you have to **borrow the radix (16)** from the next column turning the **B** into an **A**.
  - Then **B + 16 = 11 + 16 = 27_{10}** .
  - Now, **27 - D = 27 - 13 = 14 (E_h)**.

The 16's Complement System

As with Octal and Binary, we can easily subtract in the **16's Complement System** by first converting both numbers into the **16's Complement System** with the **same value of N**, finding the negative of the subtrahend, and the adding the two numbers. The rules for placing a number into the **16's Complement System** are as follows:

- Determine the number of digits you need (**N**) and pad the left side of both numbers with zero's so that the number of digits in each number total **N-1**.
  - (If you are going to perform math, all the numbers must have the same number of digits.)
- Place the **16's Complement** positive sign bit (0), followed by a **comma** on the far left of each number.
  - **The existence of a sign bit followed by a comma and the subscript 16 (or h) attached to the number together identify the entire number as being in the 16's complement system**.
- You now have a positive **16's Complement** number.

If it is desired to make one or both of the numbers negative:

- Start with the **LSB**. **Copy all zero's** up to the 1^{st} **non-zero number**.
- **When dealing with the 1^{st} non-zero number,** the number you bring down this time is **the number you would have to add to this number to get a 16**.
- For instance, if the number were a **4**, you would have to add a **12(C)** to get **16** so you would bring down a ***C***.
- Do exactly the same thing to the rest of the columns but **use 15 vice 16**.
- For example, if one of the next numbers was a **2**, you would need to **add 13 (D) to 2 to get 15**, so you would bring down a **D**.
- The resulting number is the negative of the **16's Complement** number you started with and will have an **(F,) as the negative sign bit (if you were finding the negative of a positive number)**.

16's Complement Subtraction Example

Rework the previous example, **0xBC7 – 0xDA** in the **16's Complement System:**

$$\begin{array}{rrrr} & B & C & 7_h \\ - & & D & A_h \\ \hline \end{array}$$

- First, determine the **required N value** if it is not already known. We will use an **N = 4** for this problem.
- Next, place a **16's Complement positive sign bit, 0,** followed by a **comma,** on the left side of the **BC7.**
- Next, pad the left side of the **DA_h with a zero** to bring the number of digits to **N-1=3.**
- Place a **16's Complement positive sign bit, 0,** followed by a **comma** on the left side of the **0DA.**
- Now you have two positive **16's Complement numbers** with the same **N value.**

$$\begin{array}{rrrrr} & 0, & B & C & 7_h \\ - & 0, & 0 & D & A_h \\ \hline \end{array}$$

- **Now let's find the negative of the subtrahend.** The operation is shown below.

$$\begin{array}{rrrr} 0, & 0 & D & A_h \\ \hline F, & F & 2 & 6_h \end{array}$$

  - Note that the sum of the right most column is **16** and the sum of the other columns is **15.**
- Finally we can perform the subtraction by adding the two numbers.

$$\begin{array}{rrrrr} \cancel{1} & 1 & & & \\ \hline & 0, & B & C & 7_h \\ + & F, & F & 2 & 6_h \\ \hline & 0, & A & E & D_h \end{array}$$

Note **that B + F = 26_{10}.**

16 goes into 26 one time with a remainder of A.

Also **note that the final carry, $16_{10}=10_h$ was discarded.**