Boolean (Switching) Algebra Review

Good Review book

"BeBop to the Boolean Boogie: An Unconventional Guide to Electronics", 2nd ed.

by Clive Maxwell

Hightext Publications Inc. from Amazon.com for approx. $27.14

ISBN 0750675438

Theorems and Postulates

$a + 0 = a$	$a \cdot 0 = 0$
$a + bc = (a + b)(a + c)$	
$a + \overline{a} = 1$	$a \cdot \overline{a} = 0$
$a + a = a$	$a \cdot a = a$
$a + 1 = 1$	$a \cdot 1 = a$
$\overline{\overline{a}} = a$	
$a + ab = a$	$a(a + b) = a$
$a + \overline{a}b = a + b$	$a(\overline{a} + b) = ab$
$ab + a\overline{b} = a$	$(a + b)(a + \overline{b}) = a$
$ab + a\overline{b}c = ab + ac$	$(a + b)(a + \overline{b} + c) = (a + b)(a + c)$
$\overline{a + b} = \overline{a} \cdot \overline{b}$	$\overline{a \cdot b} = \overline{a} + \overline{b}$
$ab + \overline{a}c + bc = ab + \overline{a}c$	$(a + b)(\overline{a} + c)(b + c) = (a + b)(\overline{a} + c)$

The above table is available for download in a printable version from my homepage! While you are at it, print out the "No. of 1's" document which you will need for the Quine-McCluskey topics later.

In our **review** of **Boolean Algebra**, I'll be mainly teaching by example and will be using the **Theorems and Postulates shown above.** A certain level of algebra experience is assumed, so the first six rows of rules are not going to be introduced.

Boolean Example 1 (Truth table):

Problem Statement: Use a Truth table to **Prove :** $f(a,b) = a + \bar{a}b = a + b$

a	b	$\bar{a}$	$\bar{a}b$	$a + \bar{a}b$	=	$a + b$
0	0	1	0	0	=	0
0	1	1	1	1	=	1
1	0	0	0	1	=	1
1	1	0	0	1	=	1

Q.E.D.(*Thus* it is proven)

Boolean Example 2

Problem Statement Simplify the following switching function using: $A + AB = A$

$$f(a,b,c) = \underbrace{ab + a}_{a} + a\bar{b} + a\bar{c}$$

$$= \underbrace{a + a\bar{b}}_{a} + a\bar{c}$$

$$= \underbrace{a + a\bar{c}}_{a} = a$$

$$= \sum m(4,5,6,7) = \prod M(0,1,2,3)$$

1 level circuit $\Rightarrow$
- no gates
- A WIRE!

This problem can be solved another way.

Boolean Example 3

Problem Statement: Simplify the problem from the last example by factoring 1st.

$$f(a,b,c) = ab + a + a\overline{b} + a\overline{c}$$
$$= a\underbrace{\left(b + 1 + \overline{b} + \overline{c}\right)}_{x+1=1}$$
$$= \boxed{a} \qquad \Leftarrow \text{ans}$$

Note that the result for both examples is the same.

Boolean Example 4

Problem Statement: Simplify the following switching expression.

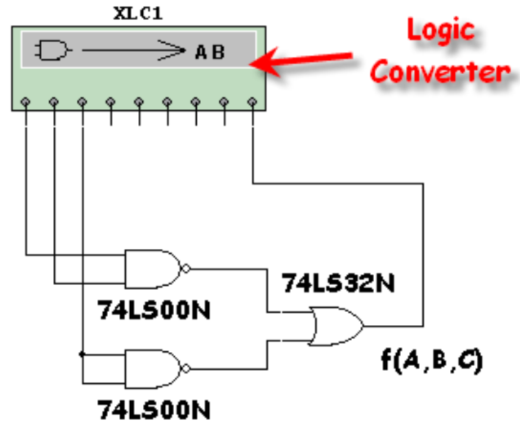
Simplify $f(a,b,c) = ac + \overline{a}bc$

- factor out the c

$$= c\left(a + \overline{a}b\right)$$
$$= c\left(a + b\right)$$
$$= \boxed{ac + bc} \qquad \Leftarrow \text{ans}$$

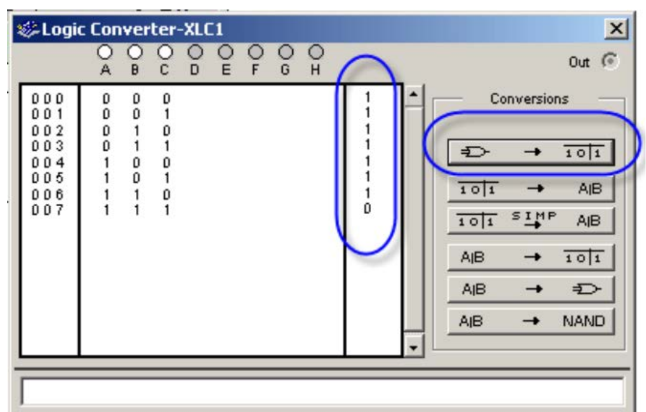
MultiSIM Example 2-1

Problem Statement: Use MultiSIMs **Logic Converter** to verify the results from the circuit shown.



Once the circuit has been built, connect the inputs and output to the Logic Converter. Be careful to insure that the inputs are connected with the MSB on the farthest input to the left. The input at the far right end is where the circuit output is connected. This device is only used with circuits which only have a single output.

Once the circuit is connected, select the "Circuit to Table" button (see above right). As can be seen, the resulting table matches the table on the previous page.



Boolean Example 5

Problem Statement: **Prove that the following Theorem is true:**

$$f(X,Y,Z) = (X+Y)(X+Z) = X + YZ$$

$$(X+Y)(X+Z) = X + YZ$$

$$\underbrace{XX}_{X} + XZ + XY + YZ = X + YZ$$

$$\underbrace{X + XZ}_{X} + XY + YZ = X + YZ$$

$$\underbrace{X + XY}_{X} + YZ = X + YZ$$

$$X + YZ = X + YZ$$

Boolean Example 6

Problem Statement: **Simplify** $f(x,y,z) = \sum m(0,3,4,5,7)$

$$\begin{aligned}
f(x,y,z) &= \bar{x}\,\bar{y}\,\bar{z} + \bar{x}yz + x\bar{y}\,\bar{z} + x\bar{y}z + xyz \\
&= \bar{x}yz + \bar{x}\,\bar{y}\,\bar{z} + x\bar{y}\,\bar{z} + x\bar{y}z + xyz \qquad \text{reorder} \\
&= \bar{x}yz + \underbrace{(\bar{x}+x)}_{(a+\bar{a}=1)}\bar{y}\,\bar{z} + x\underbrace{(\bar{y}+y)}_{(a+\bar{a}=1)}z \\
&= \bar{x}yz + \bar{y}\,\bar{z} + xz \\
&= \bar{y}\,\bar{z} + \underbrace{\underbrace{x}_{b}\underbrace{z}_{a} + \underbrace{\bar{x}}_{\bar{b}}\underbrace{y}_{c}\underbrace{z}_{a}}_{(ab+a\bar{b}c=ab+ac)} \\
&= \bar{y}\,\bar{z} + xz + yz \qquad \Leftarrow \text{ans}
\end{aligned}$$

Note the second to last line of the last example.

$$= \bar{y}\,\bar{z} + \underbrace{x}_{b}\underbrace{z}_{a} + \underbrace{\bar{x}}_{\bar{b}}\underbrace{y}_{c}\underbrace{z}_{a}$$

$$\textbf{theorem } (ab + a\bar{b}c = ab + ac)$$

$$= \bar{y}\,\bar{z} + xz + yz$$

An attempt has been made to show how that part of the expression matches one of the theorems. Note how the **(x)** has been assigned to **(b)** in the theorem and the $(\bar{X})$ has been assigned to the $(\bar{b})$ in the theorem.

Let's take a look at how this assignment might change if the problem changed slightly:

$$= \bar{y}\,\bar{z} + \underbrace{\bar{x}}_{b}\underbrace{z}_{a} + \underbrace{x}_{\bar{b}}\underbrace{y}_{c}\underbrace{z}_{a}$$

$$\textbf{theorem } (ab + a\bar{b}c = ab + ac)$$

$$= \bar{y}\,\bar{z} + \bar{x}z + yz$$

All that happened was the **(x)** and the $(\bar{X})$ traded places. Note that the assignment of **(b)** in the theorem is now made to the $(\bar{X})$ vice the **(x)** and note the change in the answer. **The student might be tempted to assign the** $(\bar{b})$ from the theorem to the $(\bar{X})$, but **this would be wrong**.

The following is an example of how an **SOP (Sum of Products)** expression can be converted into an expression which would only require **NAND** gates to implement by using **De'Morgan's Theorem**.

Boolean Example 7 (NAND ONLY LOGIC)

Problem Statement: Implement $f(x,y,z) = \bar{y}\,\bar{z} + xz + yz$ in NAND only logic.

1st, double negate both sides of the equation.

We know that $\bar{\bar{a}} = a$ so if we double negate both sides, the expression will remain UNCHANGED!

$$\overline{\overline{f(x,y,z)}} = \overline{\overline{\bar{y}\,\bar{z} + xz + yz}}$$

Once this has been done, perform DeMorgan's on the bottom negation, only applying the theorem to the signs of the expresssion

$$= \overline{\left(\overline{\bar{y}\,\bar{z}}\right)\left(\overline{xz}\right)\left(\overline{yz}\right)} \qquad \Leftarrow \text{ANSWER}$$

the resulting expression can be inplemented by using ONLY NAND gates.

The designer can use the same technique to implement a **POS (Product of Sums)** expression as a NOR only circuit.

Boolean Example 8 (NOR ONLY LOGIC)

Problem Statement: Implement the following expression in NOR only logic.

$$f(x,y,z) = (x + y + \bar{z})(\bar{y} + z)$$

$$\overline{\overline{f}}(x,y,z) = \overline{\overline{(x + y + \bar{z})(\bar{y} + z)}}$$

As before, only apply De'Morgan's to the bottom negation and then only to the signs in the expression.

$$= \overline{\overline{(x + y + \bar{z})} + \overline{(\bar{y} + z)}}$$

As noted earlier, the min-term list can easily be converted into an algebraic **SOP expression**. If a **POS expression** is desired, express the min-term list as a max-term list. Then the simplification can be carried out.

Boolean Example 9

Problem Statement: Implement and simplify the following expression in POS format.

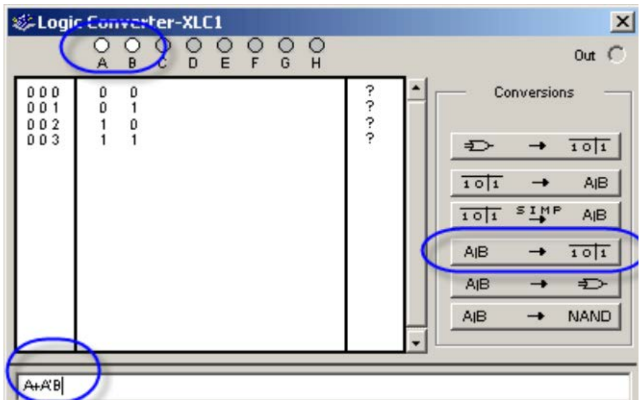
$$\begin{aligned} f(x,y,z) &= \sum m(0,3,4,5,7) \\ &= \prod M(1,2,6) \\ &= (x + y + \bar{z})\underbrace{(\underbrace{x}_{b} + \underbrace{\bar{y} + z}_{a})(\underbrace{\bar{x}}_{\bar{b}} + \underbrace{\bar{y} + z}_{a})}_{(a+b)(a+\bar{b})=a} \\ &= (x + y + \bar{z})(\bar{y} + z) \qquad \Leftarrow \textbf{Answer} \end{aligned}$$

This example also shows a simplification technique which can be very helpful. Note that the example equates $\underbrace{\bar{y} + z}_{a}$ to the **a** in the theorem $(a + b)(a + \bar{b}) = a$.

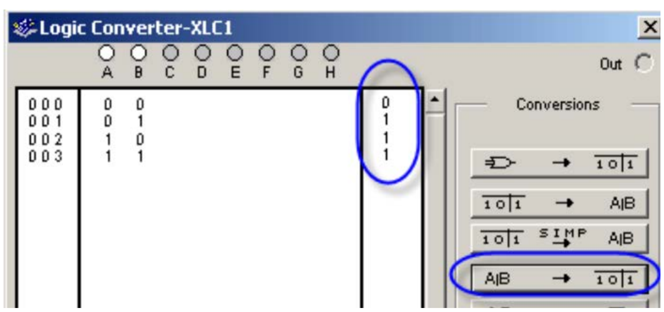
MultiSIM Example 2-2

Problem Statement: Use MultiSIMs Logic Converter to verify the proof:

Prove : $f(a,b) = a + \bar{a}b = a + b$



Open the Logic Converter and select the two bits to the left as shown above. Then type the equation in the entry area at the bottom of the device. Use a (') when a negation is required (see above right). Once the equation is complete, select the "A|B ➔ Table" button. The resulting table is shown below:



Boolean Example 10

Problem Statement: Find the minimal **POS Boolean Expression**

$$f(x,y,z) = (\bar{x}+\bar{z})(\bar{x}+y)(\bar{x}+y+z)(x+y+\bar{z})$$

reorder ***and apply two different theorems***

$$= \underbrace{\left(\underbrace{\bar{x}+y}_{a}\right)\left(\underbrace{\bar{x}+y}_{a}+\underbrace{z}_{b}\right)}_{a(a+b)=a}$$

$$\bullet \quad \underbrace{\left(\underbrace{\bar{x}}_{b}+\underbrace{\bar{z}}_{a}\right)\left(\underbrace{x}_{\bar{b}}+\underbrace{y}_{c}+\underbrace{\bar{z}}_{a}\right)}_{(a+b)(a+\bar{b}+c)=(a+b)(a+c)}$$

$$= (\bar{x}+y)(\bar{x}+\bar{z})(y+\bar{z}) \qquad \Leftarrow\Leftarrow$$

MultiSIM Example 2-3

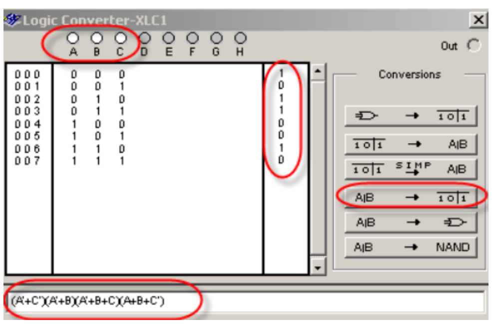
Problem Statement: Use MultiSIM's Logic Converter to verify the simplification from the previous example. In order to program this example into MultiSIM you first need to convert the (**x, y, z**) switching list into an (**A, B, C**) switching list.

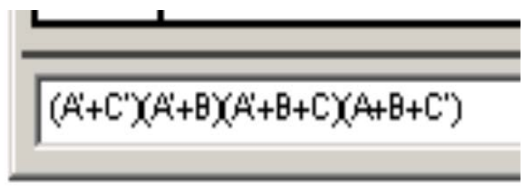
$$f(x,y,z) = (\bar{x}+\bar{z})(\bar{x}+y)(\bar{x}+y+z)(x+y+\bar{z}) \overset{?}{=} (\bar{x}+y)(\bar{x}+\bar{z})(y+\bar{z})$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$f(A,B,C) = (\bar{A}+\bar{C})(\bar{A}+B)(\bar{A}+B+C)(A+B+\bar{C}) \overset{?}{=} (\bar{A}+B)(\bar{A}+\bar{C})(B+\bar{C})$$

Next, open the Logic Converter, select the A, B, and C columns in the table and program the equation into the entry box. Don't forget to use the (') as the negation symbol.





Once you program the equation into the converter, select the "A|B ➔ Table" button. The result can be seen in the figure above.

Finally, select the $"Table \overset{simple}{\Rightarrow} A \mid B"$ button. The simplified equation will appear in the equation entry area as shown below. Note that the result is as predicted.

Boolean Example 11

Problem Statement: Simplify the following expression:

$$f(A,B,C,D) = \left(\overline{A\bar{C}}\right)\left(\overline{\bar{A}D}\right) + \left(\overline{C\bar{D}}\right)\left(\overline{\bar{B}D}\right)$$

$$= \left(\bar{A} + \underbrace{\bar{\bar{C}}}_{C}\right)\left(\underbrace{\bar{\bar{A}}}_{A} + \bar{D}\right) + \left(\bar{C} + \underbrace{\bar{\bar{D}}}_{D}\right)\left(\underbrace{\bar{\bar{B}}}_{B} + \bar{D}\right)$$

$$= \left(\bar{A} + C\right)\left(A + \bar{D}\right) + \left(\bar{C} + D\right)\left(B + \bar{D}\right)$$

$$= \underbrace{\bar{A}A}_{0} + \bar{A}\,\bar{D} + AC + C\bar{D} + B\bar{C} + \bar{C}\,\bar{D} + BD + \underbrace{D\bar{D}}_{0}$$

$$= \bar{A}\,\bar{D} + AC + \underbrace{C\bar{D} + \bar{C}\,\bar{D}}_{\bar{D}\underbrace{\left(C + \bar{C}\right)}_{1}} + B\bar{C} + BD$$

remember that any variable AND'd with "1" is equal to that variable, so $\bar{D}(1) = \bar{D}$

$$= \bar{A}\,\bar{D} + AC + B\bar{C} + \underbrace{\underbrace{B}_{b}\underbrace{D}_{\bar{a}} + \underbrace{\bar{D}}_{a}}_{\underbrace{a + \bar{a}b = a + b}_{B + \bar{D}}}$$

$$= \bar{A}\,\bar{D} + AC + B\bar{C} + B + \bar{D} \quad \text{reorder.....}$$

$$= \underbrace{\underbrace{\bar{A}}_{b}\,\underbrace{\bar{D}}_{a} + \underbrace{\bar{D}}_{a}}_{a + ab \,=\, a \,=\, \bar{D}} + AC + \underbrace{\underbrace{B}_{a}\underbrace{\bar{C}}_{b} + \underbrace{B}_{a}}_{a + ab \,=\, a \,=\, B}$$

$$= \bar{D} + B + AC \qquad \Leftarrow \textit{answer}$$

Implement as a NAND only expression

$$= \overline{\overline{\bar{D} + B + AC}} = \overline{\left(\bar{\bar{D}}\right)\left(\bar{B}\right)\left(\overline{AC}\right)}$$

$$= \overline{(D)\left(\bar{B}\right)\left(\overline{AC}\right)} \qquad \Leftarrow \text{answer}$$

MultSIM Example 2-4

Problem Statement: Verify the simplification in the previous example using Multisim.