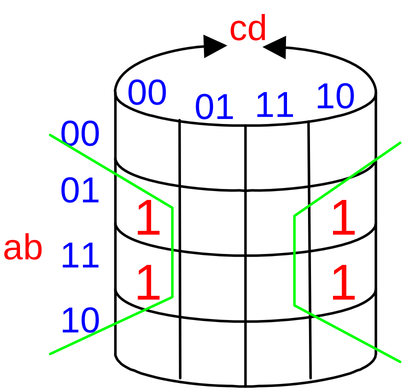
Larger K-maps

The 4-variable K-map

So far we have only discussed **2** and **3-variable K-maps**. We can now create a **4-**variable map in the same way that we created the **3-variable K-map**. This time both the columns and rows are headed by two variables. **Each side will be set up such that only one variable will change between adjacent cells.**

Examine the 4-variable k-map to the right. Note that the cells are labeled with their **min-term numbers** (table row numbers). As the min-term count counts up, the count jumps from **row 2** to **row 4** then back to **row 3.** This can be verified by noting the binary equivalent of each cell number as read from the row and column headings. **Cell 12** is the **intersection of row 11 and column 00 creating 1100$_2$,** which is a **binary 12. Cell 11** is the **intersection of row 10 and column 11 creating 1011$_2$** which is a **binary 11.**

ab \ cd	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

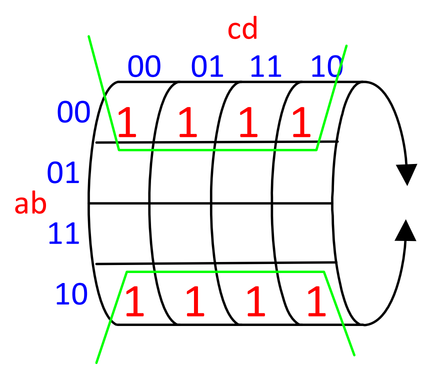


$$f(a,b,c,d) = b\overline{d}$$

The user should note that nothing else has changed. We will still **group into groups of powers of two (1, 2, 4, 8, 16, etc.) with the largest possible group in this 4-variable map at 16.**

Note in the figure to the left that if we roll this map into a cylinder along the **vertical axis**, we see that the **left** and **right edges** are still **adjacent,** just as with the **3-variable k-map**.

We can also roll the **K-map** into a cylinder along the **horizontal axis** and we can then see that the **top** and **bottom edges** are **adjacent** as well. This adjacency brings up some interesting grouping potential as is demonstrated by the following examples.



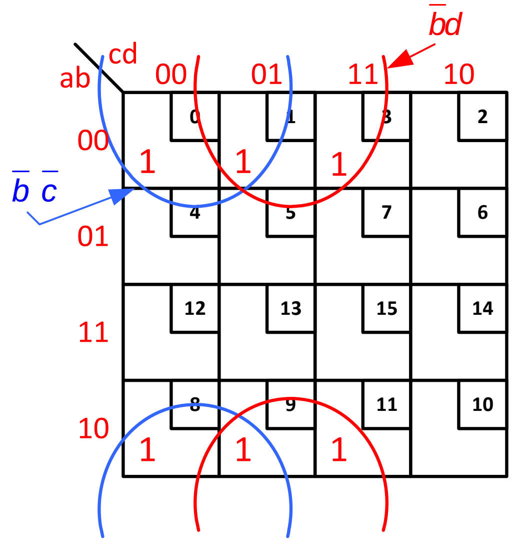
$$f(a,b,c,d) = \overline{b}$$

4-variable SOP K-map examples

4-Variable K-Map Example 1 (SOP):

Problem Statement: **Minimize the following expression into an SOP expression:**

$$f(a,b,c,d) = \sum m(0,1,3,8,9,11)$$



Due to the adjacency of the top and bottom edges, the following groups: **Groups (0,1,8,9) and (1,3,9,11)** can be created and form the minimized algebraic expression:

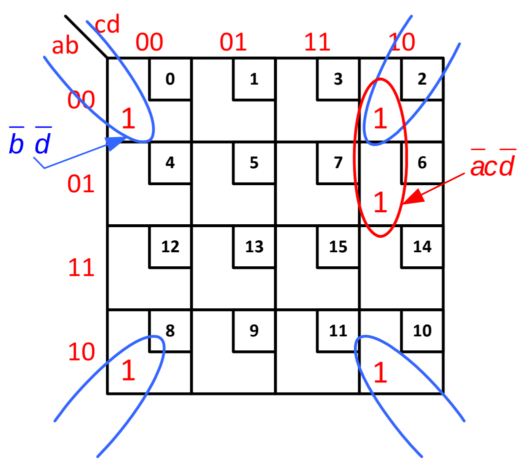
$$f(a,b,c,d) = \bar{b}\ \bar{c} + \bar{b}d$$

It should be noted how one group shared part of the other group.

4-Variable K-map Example 2 (SOP):

Problem Statement: **Minimize the following expression into an SOP expression:**

$$f(a,b,c,d) = \sum m(0,2,6,8,10)$$



This example demonstrates that the cells in the four corners make up a group of four. The second group is obvious. The minimal answer is:

$$f(a,b,c,d) = \bar{b}\ \bar{d} + \bar{a}c\bar{d}$$

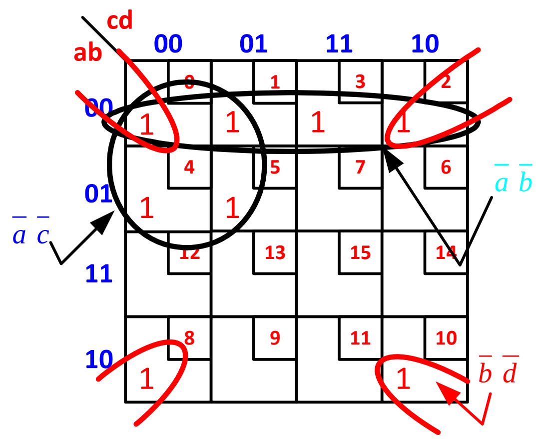
4-Variable K-Map Example 3 (SOP)

Problem Statement: **Minimize the following expression into a SOP expression:**

$$f(a,b,c,d) = \sum m(0,1,2,3,4,5,8,10)$$

The minimal solution is:

$$f(a,b,c,d) = \overline{b}\,\overline{d} + \overline{a}\,\overline{b} + \overline{a}\,\overline{c}$$



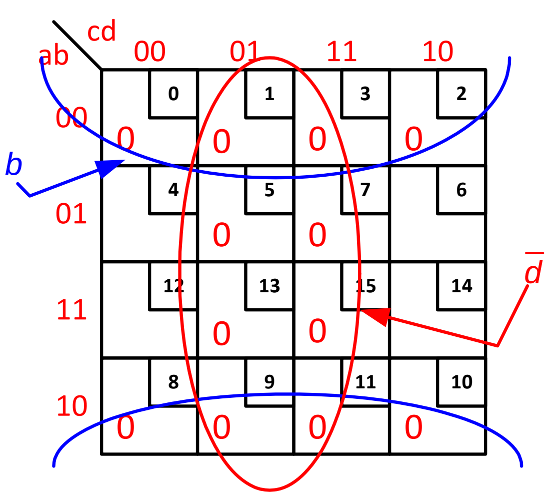
4-variable POS K-map examples

4-Variable K-Map Example 4 (POS)

Problem Statement: **Minimize the following expression into a POS expression:**

$$f(a,b,c,d) = \sum m(4,6,12,14)$$

Note that the given switching function was a min-term list and we need a max-term list for a POS expression. So, the 1st thing to do is to convert the list to a max-term list.



$$f(a,b,c,d) = \sum m(4,6,12,14)$$
$$= \prod M(0,1,2,3,5,7,8,9,10,11,13,15)$$

This example plots the **max-terms** of an expression and results in the indicated expression below. **Carefully note the logic level of the resulting terms and how they were obtained.**

$$f(a,b,c,d) = (b)(\overline{d})$$

4-Variable K-Map Example 5 (POS)

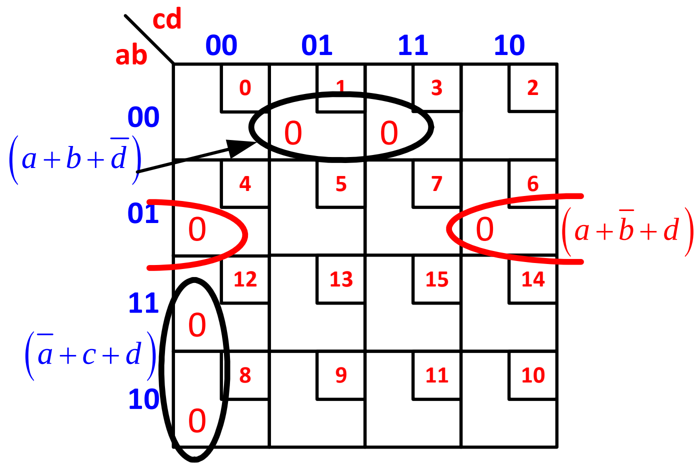
Problem Statement: **Minimize the following min-term list into a POS expression**

$$f(a,b,c,d) = \sum m(0,2,5,7,9,10,11,13,14,15)$$

Again, the provided function was in min-term form. It is necessary to convert it to a max-term list.

$$f(a,b,c,d) = \sum m(0,2,5,7,9,10,11,13,14,15)$$
$$= \prod M(1,3,4,6,8,12)$$

:



Remember that when you read off a **Max-Term**, you have to read it off like it was the inverse of a min-term.

The minimum result is:

$f(a,b,c,d) = (a+b+\bar{d})(a+\bar{b}+d)(\bar{a}+c+d)$

The Implicant, Prime Implicant (PI), and Essential Prime Implicant (EPI)

Let's discuss a couple definitions which will come in handy later.

Implicant: **Any product term (element) or group of terms.**

Prime Implicant (PI): **Largest possible term, or group of product terms, that cannot be combined with any other product term to generate a term with fewer literals than the original term.**

Essential Prime Implicant (EPI): **A Prime Implicant which must be in the final minimal answer. It is ESSENTIAL to the answer**

Now that we have these definitions, let's revisit the solution to a previous example:

$$f(A,B,C) = \overline{B}\ \overline{C} + BC + \begin{cases} \overline{A}\ \overline{B} \\ \overline{A}\ C \end{cases}$$

This solution has two different answers.

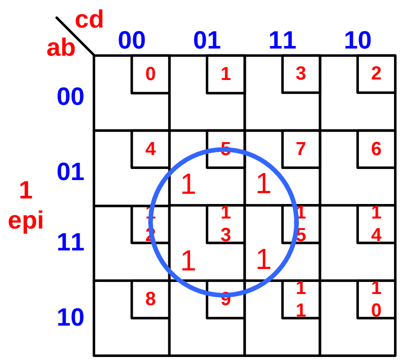
- The 1st two terms are in both answers. They are **Essential Prime Implicants (EPI)** because they are **ESSENTIAL TO THE ANSWER.**
- The second two terms are **Prime Implicants (PI)**. While one or the other is in a solution, they are **NOT ESSENTIAL to any single answer**.

Therefore, each minimal solution will consist of two **EPI's** and one **PI.**

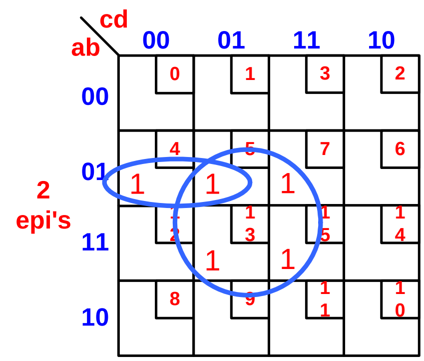
This topic will end up being used in the **Tabular Reduction Method** discussed at the end of the chapter. **While this is not a K-Map topic, these definitions are best learned in a K-Map environment**. So, let's look at a few more **K-maps** and figure out what terms are **PI's** and what terms are **EPI's.**

Essential Prime Implicant Example 1

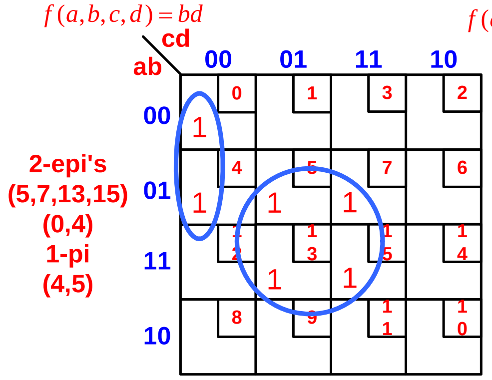
Problem Statement: Determine which terms in the K-maps below are PI's and which are EPI's.



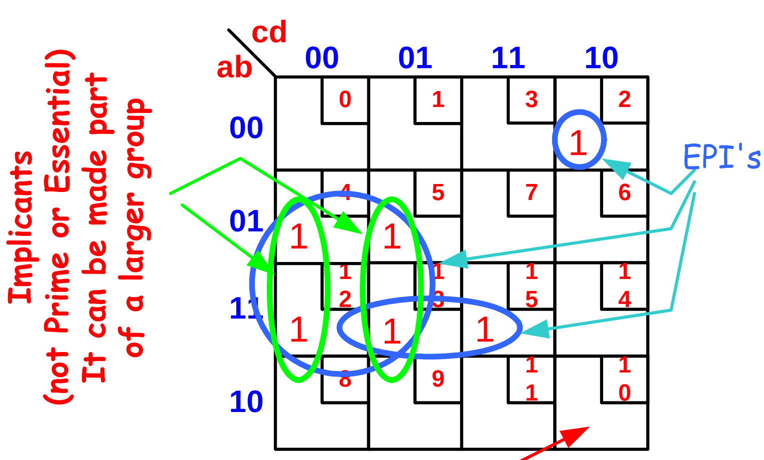
$f(a,b,c,d) = bd$



$f(a,b,c,d) = bd + \overline{a}b\overline{c}$



$f(a,b,c,d) = bd + \overline{a}\ \overline{c}\ \overline{d}$



If a '1' were added here, it would be a IMPLICANT, but not 'PRIME' because it can be made "larger." If we were to group this term with minterm 2, then it would be an EPI.

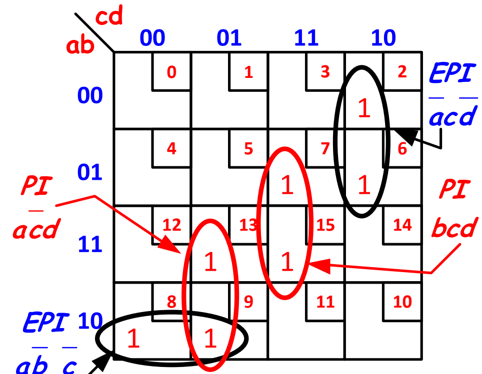
Essential Prime Implicant Example 2

Problem Statement: Determine which terms in the solution below are EPI's and PI's:

Two EPI's (2,6) and (8,9)

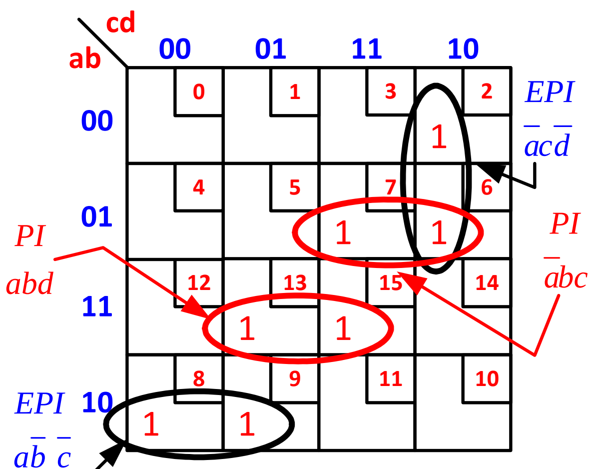
Four PI's (9,13), (7,15), (13,15), and (6,7)

Thus, this example has THREE different answers.



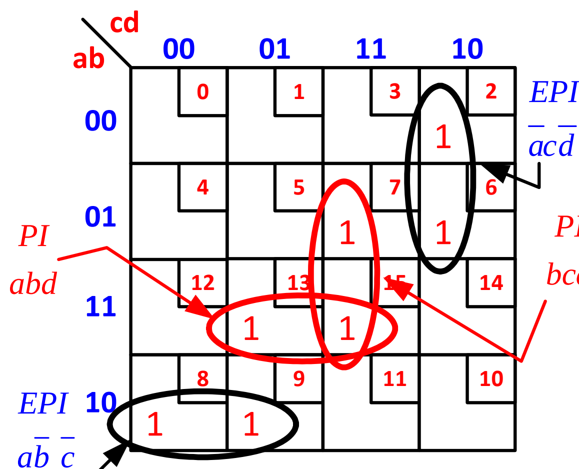
Solution #1

$$f(a,b,c,d) = \overbrace{\bar a c\bar d + a\bar b\,\bar c}^{EPI's} + \underbrace{a\bar c d + bcd}_{PI's}$$



Solution #2

$$f(a,b,c,d) = \overbrace{\bar a c\bar d + a\bar b\,\bar c}^{EPI's} + \underbrace{abd + \bar a bc}_{PI's}$$



Solution #3

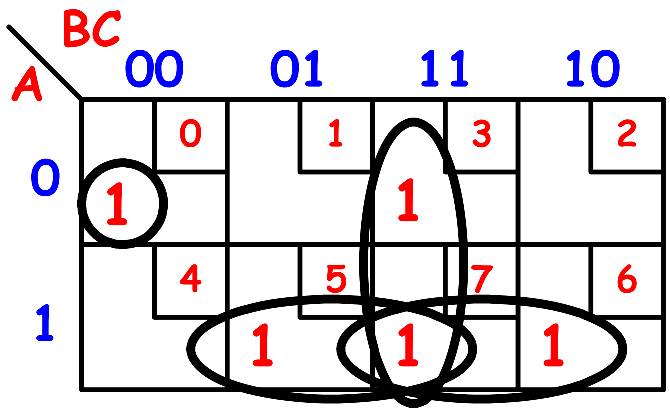
$$f(a,b,c,d) = \overbrace{\bar a c\bar d + a\bar b\,\bar c}^{EPI's} + \underbrace{abd + bcd}_{PI's}$$

$$f\left(a,b,c,d\right) = a\bar b\bar c + \bar a c\bar d + \begin{cases} bcd + a\bar c d \\ \bar a bc + abd \\ bcd + abd \end{cases}$$

Essential Prime Implicant Example 3

Problem Statement: Analyze the following expression for PI's and EPI's.

$$f(A,B,C) = \underbrace{ABC}_{7} + \underbrace{AB\overline{C}}_{6} + \underbrace{A\overline{B}C}_{5} + \underbrace{\overline{A}BC}_{3} + \underbrace{\overline{A}\,\overline{B}\,\overline{C}}_{0}$$



$$f(A,B,C) = AC + AB + BC + \overline{A}\,\overline{B}\,\overline{C}$$

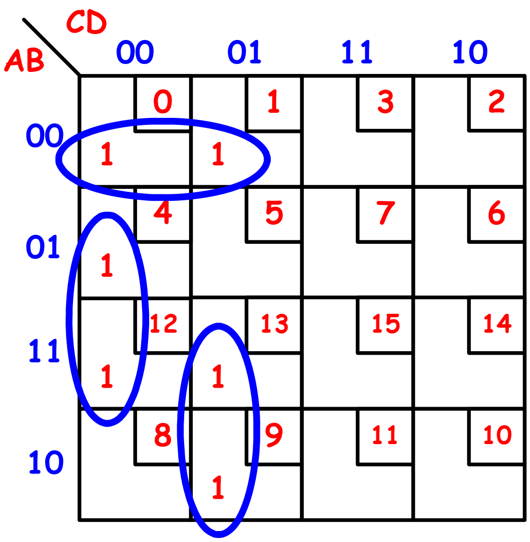
All four groups are **Prime Implicant's** because none of them can be combined with any other term to yield fewer literals. They are also all **Essential** because they each cover at least one "**1**" not covered by any other **Prime Implicant**. **Since a term is always defined by its highest definition, they are all designated as EPI's.** They are ALL ESSENTIAL TO THE ANSWER.

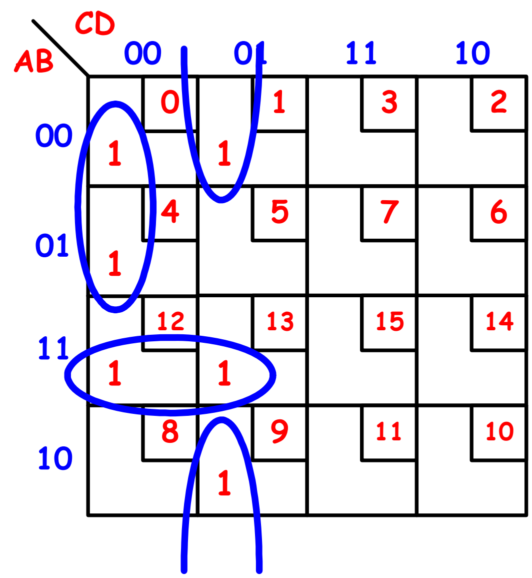
The Circular function and the Prime Implicant

Let's look at a special function known as a "**CIRCULAR Function**". A circular function is made up of <u>only</u> <u>PI's.</u> There are NO EPI's.

Circular Function Example

Problem Statement: **Examine the following K-maps to determine if the function is CIRCULAR.**



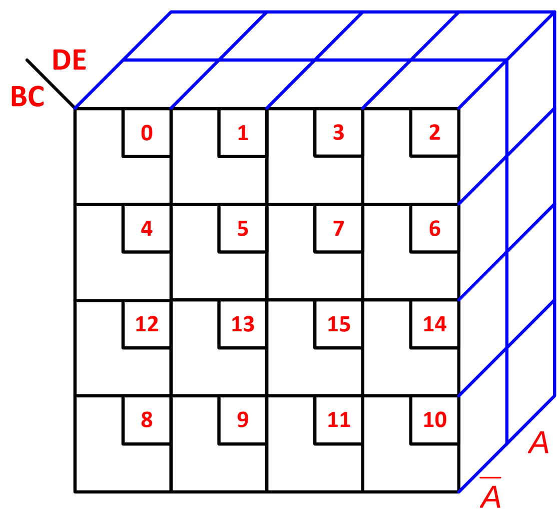


This function doesn't have any EPI's. Only 6 PI's:

(0,1), (0,4), (1,9),(4,12), (9,13), (12,13)

The 5-variable K-map

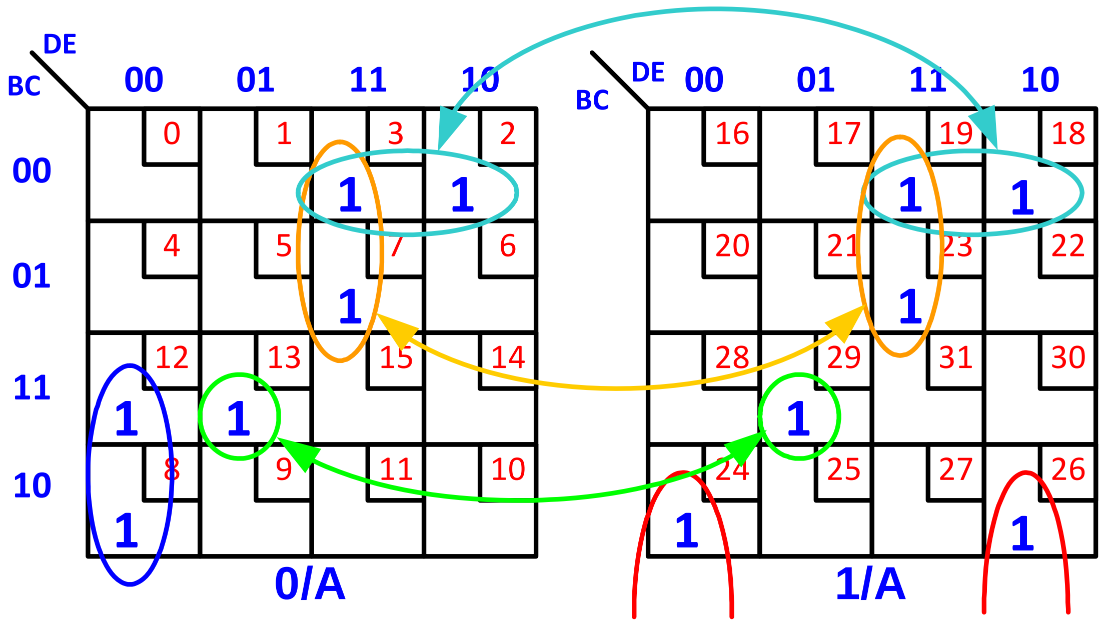
The **5-variable** K-map is three dimensional! The first 16 cells are in front and the 2nd 16 cells are in the back. Obviously we can't group things in the back half because we can't see it. So we slide out the back half and place it to the right of the front half. Note that the front half has the **MSB of '0'** ($\overline{A}$ in this example) while the back half has the **MSB of '1'** (A in this example). Of course, if this was to be used for mapping zeros instead of ones, the $\overline{A}$ would be swapped with the A. This will be a lot clearer after a few examples.



5-Variable K-Map Example 1 (SOP):

Problem Statement: Find the **minimal SOP** equation of the following expression using a 5-variable k-map.

$$f(A,B,C,D,E) = \sum m(2,3,7,8,12,13,18,19,23,24,26,29)$$



The only thing which remains is to turn the result into Boolean variables. The process below is a method that I find helpful in the larger problems.

$(2,3,18,19)$	$\bar{B}\,\bar{C}\,D$
$(3,7,19,23)$	$\bar{B}DE$
$(8,12)$	$\bar{A}\,B\,\bar{D}\,\bar{E}$
$(13,29)$	$BC\bar{D}E$
$(24,26)$	$A\,B\,\bar{C}\,\bar{E}$

$$\boxed{f(A,B,C,D,E) = \bar{B}\,\bar{C}\,D + \bar{B}\,D\,E + \bar{A}\,B\,\bar{D}\,\bar{E} + B\,C\,\bar{D}\,E + A\,B\,\bar{C}\,\bar{E}}$$

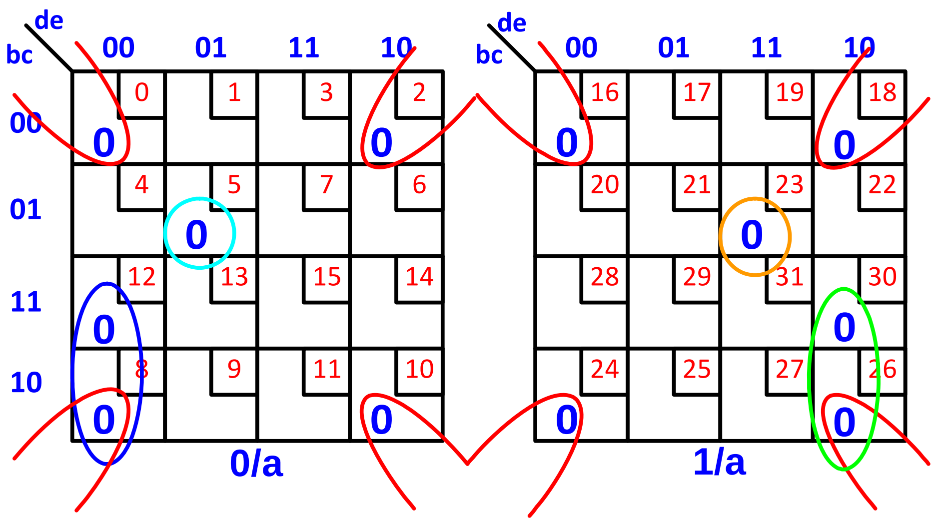
Just to review, there are 3 **EPI**'s and 2 **PI**'s in this list.

Question: Can you pick them out? PI's are (8, 12) and (24, 26)

5-Variable K-Map Example 2 (POS):

Problem Statement: Find the **MINIMAL POS** solution for the following expression.

$$f(A,B,C,D,E) = \prod M(0,2,5,8,10,12,16,18,23,24,26,30)$$



$(0,2,8,10,16,18,24,26)$	$C + E$
$(8,12)$	$A + \overline{B} + D + E$
$(26,30)$	$\overline{A} + \overline{B} + \overline{D} + E$
5	$A + B + \overline{C} + D + \overline{E}$
23	$\overline{A} + B + \overline{C} + \overline{D} + \overline{E}$

$$f(A,B,C,D,E) = (C+E)(A+\overline{B}+D+E)(\overline{A}+\overline{B}+\overline{D}+E)$$
$$(A+B+\overline{C}+D+\overline{E})(\overline{A}+B+\overline{C}+\overline{D}+\overline{E})$$

The 6 variable K-map

Just as there are 5 variable K-maps, **there are also 6 variable K-maps**. They are 4x4x4 three dimensional boxes and when expanded in the same manner as the 5 variable K-map create four boxes, two above the other two. However, I find that it is very easy to make a mistake with 6 variable K-maps. Therefore, ***I recommend that you use tabular reduction solution methods like Quine-McCluskey (discussed later) to simplify expressions which contain 6 or more variables.***