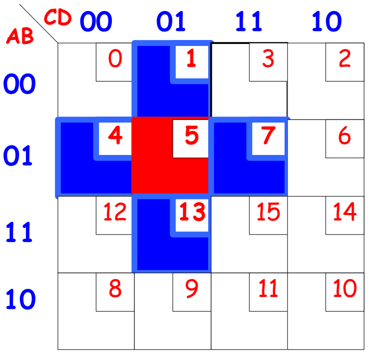
Quine-McCluskey Tabular Reduction (E)

Quine-McCluskey Introduction

- Handles any # of variables
- Does the same thing as **K-Maps** except it does it **numerically** instead of **graphically**.

Question: **Why does it work?**

Explanation: Let's examine a standard 4 x 4 **k-map**. Pick any min-term (cell); **m_5** for example.



AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Look at the cells which share an edge with **m_5**.

Question: What does each **adjacent min-term differ from m_5** by?

Answer:

$$\left.\begin{array}{llll} m1 & differs & by & (4)\ i.e\ 2^2 \\ m4 & differs & by & (1)\ i.e\ 2^0 \\ m7 & differs & by & (2)\ i.e\ 2^1 \\ m13 & differs & by & (8)\ i.e\ 2^3 \end{array}\right\} \text{all differ by powers of 2}$$

To solve an expression with Quine-McCluskey, **it must first be expressed in its numerical canonical form**. Then, each of the **min-terms** in the list is arranged into groups of terms which **share the same # of 1's in each numbers binary equivalent**.

This topic is best taught by example. Several examples follow.

Quine-McCluskey Example 1 (4-Variable SOP):

Problem Statement: **Simplify the following expression:** $f(w,x,y,z) = \sum m(0,1,2,8,11,14,15)$

- First, determine the **number of 1's** in each **min-terms** binary equivalent and write that number below the **min-term** (or **max-term**) in the list. A '**# of 1s**' handout is available on my website for you to use for larger **min-term** values.

$$f(w,x,y,z) = \sum m\left(\overset{\downarrow}{\underbrace{0}_{0}},\overset{\downarrow}{\underbrace{1}_{1}},\overset{\downarrow}{\underbrace{2}_{1}},\overset{\downarrow}{\underbrace{8}_{1}},\overset{\downarrow}{\underbrace{11}_{3}},\overset{\downarrow}{\underbrace{14}_{3}},\overset{\downarrow}{\underbrace{15}_{4}}\right)$$

- Create a column of these min-terms. The numbers have to be placed in order according to the **number of 1's** in their binary equivalent.

- Place a <u>**horizontal separator line**</u> between each group

NOTE:

There's a large X in the "2 1's" **min-term** sections since there wasn't a **min-term** with only two 1's in the list. This section **MUST** be blocked off. (Standard Student Mistake!)

THIS IS VERY IMPORTANT!!!!!!!.

# of 1's	min-terms
0	0
1	1 2 8
2	X
3	11 14
4	15

- Each cell of the table **can only be compared** with **min-terms** in the cell **directly below it.**

- **In addition, each min-term in an upper group can only be compared with a number <u>larger than it</u> is in the group directly below it. For instance: (4(1) and 3(2)), or (9(2) and 7(3)) would not be legal combinations.**

- **If they differ by a power of 2,** place them in the next column, separate them by a comma, and then place the **power of 2** they differ by next to them in **( )**. In the example:
  - **0** differs from **1** by **1** (power of 2)➔ 0,1(1)
  - **0** differs from **2** by **2** (power of 2)➔ 0,2(2)
  - **0** differs from **8** by **8** (power of 2)➔ 0,8(8)
  - **If a min-term is used in a group, a check-mark is placed next to it. It can still be used, but if it is checked, it is not a PI.** If an entry <u>isn't checked off</u>, **it is considered to be a PI.** A single **min-term** in the 1^{st} column is analogous to a single **1** in a **K-map** grouped by itself.

min-terms	groups of 2
0 √	X
1 √ 2 √ 8 √	0,1(1) 0,2(2) 0,8(8)
X	
11 14	
15	

- If all possible groups have been found with **min-term 0**, **a horizontal line is drawn**, and you go to the next set of **min-terms**.

- We want to group the **1**,**2**, and **8** with the next group down. In this case, since the next group down from them is empty, place an X in the '**groups of 2s**' block for that set. **There can't be any comparisons**.

min-terms	groups of 2
0 √	X
1 √ 2 √ 8 √	0,1(1) 0,2(2) 0,8(8)
X	X
11 14	
15	

Example continued)

- Now jump to the next set of **min-terms**, **11** and **14**.
  - **11** differs from **15** by **4** (power of 2) ➔ 11,15(4)
  - **14** differs from **15** by **1** (power of 2) ➔ 14,15(1)

- At this point, all **min-terms** in the column have been attempted, so before moving on, check to see if there is a **min-term** which has not been checked off. If there is one,
  - Check to see if you made a mistake, then
  - If no mistakes, **circle it: It will be a PI (actually, circles in this column will end up as EPIs). In this case, all of them have been checked off.**

min-terms	groups of 2
0 √	X
1 √ 2 √ 8 √	0,1(1) 0,2(2) 0,8(8)
X	X
11 √ 14 √	11,15(4) 14,15(1)
15 √	

- This example has been grouped as far as it can go. We know that this time because there are no **ADJACENT sections in column 2**. They have **X**'s between them. So, any group in the column which does not have a **check-mark** (all of them) gets circled (or identified in any other obvious manner) to indicate that they are **PI**'s.

At this point we would normally set up a table **(PI TABLE)** to determine:

- Which terms are **EPI**'s and
- Which terms are **PI**'s.
- Then we would determine which of the **PI**'s to use for the **minimal solution**.

However, in this case, they are all **EPI's,** so we will hold off on introducing the **PI Table** for later.

Returning the min-terms to their Boolean Variable Equivalent

- Set up a matrix with the **PIs** and **EPIs** resulting from the **PI Table.**
  - In this case, since there wasn't a **PI Table**, use the circled values.
- Place the values down the left side, and the
- Variables from the switching list along the top. The variables <u>**HAVE** to be in order</u> with the **MSB** on the left and the **LSB** on the right.
- Place the **Powers of 2** (… **16, 8, 4, 2, 1**) above the associated variable.

	8	4	2	1	
	w	**x**	**y**	**z**	**Boolean**
0,1(1)					
0,2(2)					
0,8(8)					
11,15(4)					
14,15(1)					

Example Continued)

Let's look at the 1st pair, **0,1 (1):**

- Choose either the 0 or the 1 (it doesn't matter which one). In this case we chose the '1'.
    - Write its binary equivalent below the switching variables.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	1	
0,2(2)					
0,8(8)					
11,15(4)					
14,15(1)					

- Now look at the value in **(1)** which is the amount the 2 numbers differed by. That is a **power-of-two**. Replace the bit in that position with an X or a **-**.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	–	
0,2(2)					
0,8(8)					
11,15(4)					
14,15(1)					

- Now write the **Boolean** algebra equivalent for each bit that is left in the **Boolean** column. Since this is an **SOP** expression (the list was a list of **min-terms**), then they should be AND'ed together.
- The next two have been done as well.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	–	$\overline{w}\,\overline{x}\,\overline{y}$
0,2(2)	0	0	–	0	$\overline{w}\,\overline{x}\,\overline{z}$
0,8(8)	–	0	0	0	$\overline{x}\,\overline{y}\,\overline{z}$
11,15(4)					
14,15(1)					

- Now look at the **11, 15 (4)** group.
    - Choose one of them (it doesn't matter which), and place its binary equivalent below the switching variables.
    - Replace the 0 in the '**8**' column with a '-' and then convert the expression to its **Boolean** Equivalent. The last two have been finished for you.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	–	$\overline{w}\,\overline{x}\,\overline{y}$
0,2(2)	0	0	–	0	$\overline{w}\,\overline{x}\,\overline{z}$
0,8(8)	–	0	0	0	$\overline{x}\,\overline{y}\,\overline{z}$
11,15(4)	1	–	1	1	**wyz**
14,15(1)	1	1	1	–	**wxy**

- Finally, since this is an **SOP** expression, put each of the **EPI** terms together with '**OR**' operators and you have found the **minimum solution** for the given expression.

$$f(w,x,y,z) = \overline{w}\;\overline{x}\;\overline{y} + \overline{w}\;\overline{x}\;\overline{z} + \overline{x}\;\overline{y}\;\overline{z} + wyz + wxy$$

Quine-McCluskey Example 2 (4-Variable SOP):

Problem Statement: **Repeat the last example, but this time, add min-term 10 to the list.**

$$f(w,x,y,z)=\sum m\left(\overset{\downarrow}{\underbrace{0}_{0}},\overset{\downarrow}{\underbrace{1}_{1}},\overset{\downarrow}{\underbrace{2}_{1}},\overset{\downarrow}{\underbrace{8}_{1}},\overset{\downarrow}{\underbrace{10}_{2}},\overset{\downarrow}{\underbrace{11}_{3}},\overset{\downarrow}{\underbrace{14}_{3}},\overset{\downarrow}{\underbrace{15}_{4}}\right)$$

- The **min-term** list with the additional min-term is shown above. The # of 1's has been performed on the list.
- The **min-terms** have been placed into '**# of 1s**' order to the right:

# of 1's	min-terms
0	0
	1
1	2
	8
2	10
3	11
	14
4	15

The solution has been completed up to the point of the use of the new **min-term** to the right:

- This time the grouping of **1, 2,** and **8** can be attempted with **min-term 10.**
  - o **1** differs from **10** by **9** (FAILED)
  - o **2** differs from **10** by **8** (power of 2)➔ 2,10(8)
  - o **8** differs from **10** by **2** (power of 2)➔ 8,10(2)
  - o A horizontal line has been placed after the last group.
  - o A **check-mark** has been added to **min-term 10** since it has been used in a group. **It will NOT be a PI.**

min-terms	groups of 2
0 √	X
1 √	0,1(1)
2 √	0,2(2)
8 √	0,8(8)
10 √	2,10(8)
	8,10(2)
11	
14	
15	

- Now the grouping of **min-term 10** can be attempted with **min-terms 11** and **14**:
  - o **10** differs from **11** by **1** (power of 2)➔ 10,11(1)
  - o **10** differs from **14** by **4** (power of 2)➔ 10,14(4)
  - o A horizontal line has been placed after the last group.
  - o A **check-mark** has been added to **min-terms 11 and 14** since they have been used in a group. **They are NOT PIs.**

min-terms	groups of 2
0 √	X
1 √	0,1(1)
2 √	0,2(2)
8 √	0,8(8)
10 √	2,10(8)
	8,10(2)
11 √	10,11(1)
14 √	10,14(4)
15	

- Finally, grouping of **min-terms 11** and **14** can be attempted with **min-term 15**:
  - o **11** differs from **15** by **4** (power of 2)➔ 11,15(4)
  - o **14** differs from **15** by **1** (power of 2)➔ 14,15(1)
  - o A **check-mark** has been added to **min-terms 15** since it has been used in a group. **It is NOT a PI.**

min-terms	groups of 2
0 √	X
1 √	0,1(1)
2 √	0,2(2)
8 √	0,8(8)
10 √	2,10(8)
	8,10(2)
11 √	10,11(1)
14 √	10,14(4)
15 √	11,15(4)
	14,15(1)

- At this point, all **min-terms** in the column have been attempted, so before moving on, check to see if there is a **min-term** which has not been checked off. If there is one,
  - o Check to see if you made a mistake, then if
  - o If no mistakes, **circle it. It will be a PI (actually, circles in this column will end up as EPIs). In this case, all have been checked off.**

Example Continues)

This is where things get real different from the 1st example. Since the **groups of terms are now adjacent**, they have the opportunity to be grouped together. So, we will potentially have a 'groups of 4' column.

To recap the grouping rules, when we **are looking for differences between groups**, we still:

- Only compare groups from which are **adjacent**. So, the **0,1(1), 0,2(2), 0,8(8)** sets can only be compared with the **adjacent** sets **2,10(8)** and **8,10(2)**.
- The upper number must be smaller than the lower number.
- We either compare the 1st number in each pair or the second number in each pair.

min-terms	groups of 2
0 √	╳
1 √ 2 √ 8 √	0,1(1) 0,2(2) 0,8(8)
10 √	2,10(8) 8,10(2)
11 √ 14 √	10,11(1) 10,14(4)
15 √	11,15(4) 14,15(1)

Now we add an additional grouping rule:

- **You can only compare groups that share the same number in the ( ).**

So, let's start grouping.

- We 1st focus on the **0,1(1)** group. **We can only compare it with a group in the adjacent set which also has a (1).** In this case, one does not exist.
    - Circle the **0,1(1)**. <u>It can't be grouped</u> and is a **PI**.
- We now focus on the **0,2(2)**. There is only one group with a **(2), [8,10(2)]**, in the adjacent set so we compare the **<u>1st number in each group</u>**.
    - **0** differs from **8** by **8** (power of 2) ➔ 0,2,8,10 (2,8)
        - **This time, we write the two groups together and follow them with**
            - **The original number in the ( ) which enabled them to be compared in the 1st place, (2), then we add in the new value that the two groups differ by (8).**
        - We place a **check-mark** beside both groups indicating that they have been used to make up a group. They will **NOT be PIs**.

min-terms	groups of 2	groups of 4
0 √	╳	
1 √ 2 √ 8 √	0,1(1) 0,2(2) √ 0,8(8)	0,2,8,10 (2,8)
10 √	2,10(8) 8,10(2) √	
11 √ 14 √	10,11(1) 10,14(4)	
15 √	11,15(4) 14,15(1)	

Example Continues)

- We now focus on the **0,8(8)**. There is only group in the next set which shares the **(8)**, the **2,10(8)** group. So,
  - **0** differs from **2** by **2** (power of 2)
    - → 0,8,2,10 (8,2)
  - We place a **check-mark** next to both groups indicating that they have been used to make up a new group. They will **NOT be PIs**.
- Draw a **horizontal line** to separate this '**group of 4**' set from the next set.

min-terms	groups of 2	groups of 4
0 √	X	
1 √ 2 √ 8 √	0,1(1) 0,2(2) √ 0,8(8) √	0,2,8,10 (2,8) 0,8,2,10 (8,2)
10 √	2,10(8) √ 8,10(2) √	
11 √ 14 √	10,11(1) 10,14(4)	
15 √	11,15(4) 14,15(1)	

Before we go any further, **each term in the 'groups of 4' column will have a DUPICATE**. For instance, look at the two groups that are there now:

- Every number in the **0,2,8,10 (2,8)** is repeated in the **0,8,2,10 (8,2)** group, just in a different order.
- A group's **duplicate** is not necessarily in the same set as its mate. It might be further down.
- If a group in this column does not have a duplicate, **THEN YOU HAVE MADE A MISTAKE!!!!**
  - Choose the group with the **numbers which are out of order** and **line it out**. In this case, **line out the**
    - **0,8,2,10 (8,2)**

min-terms	groups of 2	groups of 4
0 √	X	
1 √ 2 √ 8 √	0,1(1) 0,2(2) √ 0,8(8) √	0,2,8,10 (2,8) ~~0,8,2,10~~ ~~(8,2)~~**Duplicate**
10 √	2,10(8) √ 8,10(2) √	
11 √ 14 √	10,11(1) 10,14(4)	
15 √	11,15(4) 14,15(1)	

Example Continued)

Now let’s shift our focus to the next set. We will compare the **2,10(8)** and 8,10(2) groups with the 2 groups in adjacent set.

min-terms	groups of 2	groups of 4
0 √	X	
1 √ 2 √ 8 √	$\boxed{0,1(1)}$ 0,2(2) √ 0,8(8) √	0,2,8,10 (2,8) ~~0,8,2,10~~ ~~(8,2)~~**Duplicate**
10 √	2,10(8) √ 8,10(2) √	X
11 √ 14 √	10,11(1) 10,14(4)	
15 √	11,15(4) 14,15(1)	

- The **(8)** does not have a match in the next set. However, it has a **check-mark** so we leave it alone.
- The **(2)** does not have a match in the next set. It also has a check-mark so it is left alone as well.
- An **X** is placed in the “**Group of 4**’ column location where any matches would have been placed as a **place holder** and we move on to the next sets.

Finally we focus on the **10,11(1)** and **10,14(4)** set comparison with the **11,15(4)** and **14,15(1)** set.

min-terms	groups of 2	groups of 4
0 √	X	
1 √ 2 √ 8 √	$\boxed{0,1(1)}$ 0,2(2) √ 0,8(8) √	0,2,8,10 (2,8) ~~0,8,2,10~~ ~~(8,2)~~**Duplicate**
10 √	2,10(8) √ 8,10(2) √	X
11 √ 14 √	10,11(1) √ 10,14(4) √	10,11,14,15 (1,4) ~~10,14,11,15~~ ~~(4,1)~~**Duplicate**
15 √	11,15(4) √ 14,15(1) √	

- The **(1)** with the **10,11** group has a match in the **14,15(1)**. Comparing the 1st # in each group we find that:
  - o 10 differs from **14** by **4** (power of 2)
    - ➔ **10,11,14,15(1,4)**
  - o A **check-mark** is added next to each group. **They are NOT PIs.**
- The **(4)** with the **10,14** group has a match with the **11,15(4)** group. Comparing them we get:
  - o 10 differs from **11** by **1** (a power of 2)
    - ➔ **10,14,11,15 (4,1)**
  - o A **check-mark** is added next to each group. **They are NOT PIs.**
- Finally, we check for **duplicates** and **line out the duplicate** which has its numbers out of order.

Example Continues)

Before moving on, scan the column and check for any groups which do not have check-marks. If there is one or more of them then:

- Check to see if you made and error.
- If no error, circle it. It will be a PI.

In this case, we already have accounted for the **single PI** in the column.

Shifting our focus to the next column, we note that here are **NO ADJACENT sets**. Therefore it will not be necessary to create a new column (**Group of 8**).

Since there will be no further grouping, circle the remaining terms in that column. **They are PI's.**

min-terms	groups of 2	groups of 4
0 √	X	
1 √ 2 √ 8 √	0,1(1) 0,2(2) √ 0,8(8) √	0,2,8,10(2,8) ~~0,8,2,10~~ ~~(8,2)~~**Duplicate**
10 √	2,10(8) √ 8,10(2) √	X
11 √ 14 √	10,11(1) √ 10,14(4) √	10,11,14,15(1,4) ~~10,14,11,15~~ ~~(4,1)~~**Duplicate**
15 √	11,15(4) √ 14,15(1) √	

Again, I know that **ALL 3 of these PIs are EPI's** so this would not be a good time to introduce the **PI TABLE.** So, we will go directly to converting the PI's to their Boolean equivalents to get the minimum Boolean expression.

- Create the matrix as before. The **PIs** are on the left and the **switching variables** with their **powers of 2** are along the top.

	8	4	2	1	
	w	**x**	**y**	**z**	**Boolean**
0,1(1)					
0,2,8,10(2,8)					
10,11,14,15(1,4)					

- Choose one number from each group and **write its binary equivalent** beneath the **switching variables**.

	8	4	2	1	
	w	**x**	**y**	**z**	**Boolean**
0,1(1)	0	0	0	1	
0,2,8,10(2,8)	1	0	1	0	
10,11,14,15(1,4)	1	1	1	1	

Example Continues)

- Replace the bit(s) in each row by Xs or some other mark in the position pointed to by the numbers in the ( ) in each row.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	–	
0,2,8,10(2,8)	–	0	–	0	
10,11,14,15(1,4)	1	–	1	–	

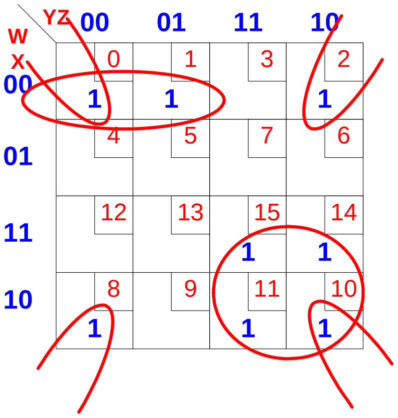
- Convert each term to its Boolean equivalent.

	8	4	2	1	
	w	x	y	z	**Boolean**
0,1(1)	0	0	0	–	$\overline{w}\,\overline{x}\,\overline{y}$
0,2,8,10(2,8)	–	0	–	0	$\overline{x}\,\overline{z}$
10,11,14,15(1,4)	1	–	1	–	$w\,y$

- Put them together with **OR** operators and you have the **minimal solution** shown below:

$$\boxed{f(w,x,y,z)=\overline{x}\;\overline{z}+w\;y+\overline{w}\;\overline{x}\;\overline{y}}$$

Let's now look at the same example using a **K-map**. You will note that the result for this **K-map** is the **same as for the Q-M Tabular reduction.**



Quine-McCluskey Example 3 (4-Variable SOP):

Problem Statement: **Minimize the following SOP expression using Quine-McCluskey.**

$$f(A,B,C,D) = \Sigma m\left(\underbrace{2}_{1},\underbrace{4}_{1},\underbrace{6}_{2},\underbrace{8}_{1},\underbrace{9}_{2},\underbrace{10}_{2},\underbrace{12}_{2},\underbrace{13}_{3},\underbrace{15}_{4}\right)$$

This is basically a repeat of the last examples methods up until now. The work has been done for you since the purpose of this example is to introduce the **PI Table**.

2 ✔ 4 ✔ 8 ✔	2, 6 (4) ☆PI2 2, 10 (8) ☆PI3 4, 6 (2) ☆PI4 4, 12 (8) ☆PI5 8, 9 (1) ✔ 8, 10 (2) ☆PI6 8, 12 (4) ✔	8, 9, 12, 13 (1,4) ☆PI1 ~~8, 12, 9, 13 (4,1)~~ DUPE
6 ✔ 9 ✔ 10 ✔ 12 ✔		All the ☆'s are prime implicant
13 ✔	9, 13 (4) ✔ 12,13 (1) ✔	
15 ✔	13, 15 (2) ☆PI7	

Unlike the last two examples, we do not know if the PIs found in the Tabular Reduction are **Essential" Prime Implicant's or just Prime Implicant's.** To find out which terms need to be selected to have a minimal solution, we need to create what is known as a **Prime Implicant Table**.

The Prime Implicant Table

When setting up the table, list the identified **Prime Implicant's down the left side of the table** and the **min-terms in the list along the top of the table** as shown to the right.

Min – terms ↘ PIs ⇓	2	4	6	8	9	10	12	13	15
2,6(4)									
2,10(8)									
4,6(2)									
4,12(8)									
8,10(2)									
13,15(2)									
8,9,12,13(1,4)									

Example Continues)

Once the table has been set up, **place an X in the column of each min-term which is covered by a Prime Implicant**.

- For instance, the **2,6(4) PI** will have **X**'s in the **2** and the **6 min-term** columns.
- **PI - 8,9,12,13(1,4)** should have **X**'s in **min-term** columns **8, 9, 12, 13**.
- **I call this "taking inventory."**

Min – terms ↘ **PIs** ⇓	2	4	6	8	9	10	12	13	15
2,6(4)	X		X						
2,10(8)	X					X			
4,6(2)		X	X						
4,12(8)		X					X		
8,10(2)				X		X			
13,15(2)								X	X
8,9,12,13(1,4)				X	X		X	X	

The Search for the "LONELY X"!

- The next step is to find **min-term columns** which have a **"single X"** in them. I call these X's, **"lonely X's."**
- Circle each "**LONELY X**" as it is found.

	Min – terms ↘ **PIs** ⇓	2	4	6	8	9	10	12	13	15
	2,6(4)	X		X						
	2,10(8)	X					X			
	4,6(2)		X	X						
	4,12(8)		X					X		
	8,10(2)				X		X			
EPI	13,15(2)								X	☒
EPI	8,9,12,13(1,4)				X	☒		X	X	

There are two columns with **Lonely X's**. The **PI**s in those two rows are **Essential Prime Implicant's**. Since they are the **ONLY PI's** which cover these two **min-terms**, they are **ESSETIAL to the answer**!

Example Continued)

The next step is to **mark each min-term which is covered by an EPI**.

- For instance, in this example, **EPI 13,15(2) covers min-terms 13** and **15**. I placed a **check-mark** above each of them.
- Of course, there is one more **EPI** (see below)

	Min – terms ↘								√	√
	PIs ⇓	2	4	6	8	9	10	12	13	15
	2,6(4)	X		X						
	2,10(8)	X					X			
	4,6(2)		X	X						
	4,12(8)		X					X		
	8,10(2)				X		X			
EPI	13,15(2)								X	$\boxed{X}$
EPI	8,9,12,13(1,4)				X	$\boxed{X}$		X	X	

- **EPI 8,9,12,13(1,4) covers min-terms 8, 9, 12,** and **13**. I have placed a **check-mark** above each of these columns.
- The next thing to do is the circle all min-terms which are not covered by an EPI (don't have check-marks above them).

	Min – terms ↘				√	√		√	√	√
	PIs ⇓	$\boxed{2}$	$\boxed{4}$	$\boxed{6}$	8	9	$\boxed{10}$	12	13	15
	2,6(4)	X		X						
	2,10(8)	X					X			
	4,6(2)		X	X						
	4,12(8)		X					X		
	8,10(2)				X		X			
EPI	13,15(2)								X	$\boxed{X}$
EPI	8,9,12,13(1,4)				X	$\boxed{X}$		X	X	

Example Continues)

The final step in the **PI table** is to select the:

- **Smallest # of PIs** while
- Selecting the **largest groups 1st**.

which cover ALL OF these circled, uncovered min-terms.

There might be more than one answer. In this case, there is only one set of choices: **PIs 2,10 and 4,6.**

	Min – terms ↘				√	√		√	√	√
	PIs ⇓	$\boxed{2}$	$\boxed{4}$	$\boxed{6}$	8	9	$\boxed{10}$	12	13	15
	2,6(4)	X		X						
PI	2,10(8)	X					X			
PI	4,6(2)		X	X						
	4,12(8)		X					X		
	8,10(2)				X		X			
EPI	13,15(2)								X	$\boxed{X}$
EPI	8,9,12,13(1,4)				X	$\boxed{X}$		X	X	

So, our final answer will be made up of two EPI's and two PI's. The final step in the example is to convert these terms into a minimal Boolean expression. This is done in the same manner as in the previous two examples.

		8	4	2	1	
		A	B	C	D	**Boolean**
EPI	8,9,12,13(1,4)	1	–	0	–	$A\overline{C}$
EPI	13,15(2)	1	1	–	1	ABD
PI	2,10(8)	–	0	1	0	$\overline{B}C\overline{D}$
PI	4,6(2)	0	1	–	0	$\overline{A}B\overline{D}$

The final <u>**minimal solution**</u> to this problem is:

$$f(A,B,C,D) = A\overline{C} + ABD + \overline{B}C\overline{D} + \overline{A}B\overline{D}$$

This is an example of a problem where there aren't any **EPI**'s. There are only **PI**'s. One representative of this type of problem is the **Circular Function**.

Quin-McCluskey Example 4 (Circular Function): Nelson Prob. 3-58B

Problem Statement: ***Simplify the following expression using Quine-McCluskey Tabular Reduction.***

$$f(A,B,C,D)=\sum m\left(\underbrace{0}_{0},\underbrace{2}_{1},\underbrace{4}_{1},\underbrace{5}_{2},\underbrace{10}_{2},\underbrace{11}_{3},\underbrace{13}_{3},\underbrace{15}_{4}\right)$$

Each min-term has already been classified as to how many 1's it has in it in the expression above. The problem will be solved for you in a lot fewer steps. I recommend you try to solve it on your own before you look at the answer.

When the problem is solved, the only terms are in the 'groups of two' column. The problem could not go into a 3rd column due to the rule about:

- **only being able to compare groups which share the same number in the ( ).**

So, we are left with **8 PI's** to use to determine the final minimal solution. The obvious next step is to create the PI Table.

min-terms	groups of 2
0 √	0,2 (2) **PI** 1
	0,4 (4) **PI** 2
2 √	2,10 (8) **PI** 3
4 √	4,5 (1) **PI** 4
5 √	5,13 (8) **PI** 5
10 √	10,11 (1) **PI** 6
11 √	11,15 (4) **PI** 7
13 √	13,15 (2) **PI** 8

Prime Implicant Table

Min – terms ↘ **PIs ⇓**	0	2	4	5	10	11	13	15
0,2(2)	X	X						
0,4(4)	X		X					
2,10(8)		X			X			
4,5(1)			X	X				
5,13(8)				X			X	
10,11(1)					X	X		
11,15(4)						X		X
13,15(2)							X	X

When the PI table was filled out, it was found that there weren't any **Lonely X's**. Therefore, **there weren't any EPI's** and there weren't any min-terms covered by those EPI's. Therefore, all the min-terms in the list were circled.

The chore here will be to find the:

- *Smallest number of PI's using the*
- *Largest groups 1st*

That will totally cover all the min-terms.

As indicated at the beginning of the problem is that this is a **Circular Function**. It will have several answers.

One such answer which covers all min-terms is shown here.

Prime Implicant Table

	Min – terms ↘	√	√	√	√	√	√	√	√
	PIs ⇓	$\boxed{0}$	$\boxed{2}$	$\boxed{4}$	$\boxed{5}$	$\boxed{10}$	$\boxed{11}$	$\boxed{13}$	$\boxed{15}$
PI	0,2(2)	X	X						
	0,4(4)	X		X					
	2,10(8)		X			X			
PI	4,5(1)			X	X				
	5,13(8)				X			X	
PI	10,11(1)					X	X		
	11,15(4)						X		X
PI	13,15(2)							X	X

Prime Implicant Table

	Min – terms ↘	√	√	√	√	√	√	√	√
	PIs ⇓	$\boxed{0}$	$\boxed{2}$	$\boxed{4}$	$\boxed{5}$	$\boxed{10}$	$\boxed{11}$	$\boxed{13}$	$\boxed{15}$
	0,2(2)	X	X						
PI	0,4(4)	X		X					
PI	2,10(8)		X			X			
	4,5(1)			X	X				
PI	5,13(8)				X			X	
	10,11(1)					X	X		
PI	11,15(4)						X		X
	13,15(2)							X	X

This is **another minimal solution** for the same problem. It is equivalent to the 1st one since it has the same number of PIs which are the same size as in the 1st solution.

Solution #1

Selected		8	4	2	1	
PI's↓		**A**	**B**	**C**	**D**	**Boolean**
0,2	(2)	0	0	–	0	$\overline{A}\ \overline{B}\ \overline{D}$
4,5	(1)	0	1	0	–	$\overline{A}B\overline{C}$
10,11	(1)	1	0	1	–	$A\overline{B}C$
13,15	(2)	1	1	–	1	ABD

$f(A,B,C,D)=\overline{A}\ \overline{B}\ \overline{D}+\overline{A}B\overline{C}+A\overline{B}C+ABD$

⇐ **or** ⇒

Solution #2

Selected		8	4	2	1	
PI's↓		**A**	**B**	**C**	**D**	**Boolean**
0,4	(4)	0	–	0	0	$\overline{A}\ \overline{C}\ \overline{D}$
2,10	(8)	–	0	1	0	$\overline{B}C\overline{D}$
5,13	(8)	–	1	0	1	$B\overline{C}D$
11,15	(4)	1	–	1	1	ACD

$f(A,B,C,D)=\overline{A}\ \overline{C}\ \overline{D}+\overline{B}C\overline{D}+B\overline{C}D+ACD$

As was mentioned earlier in the course, if there is more than one minimal answer, you are required to show 2 of them. Care must be taken in how you do so. This one is interesting in that neither answer shares any terms with the other.

The **minimal answer** is:

$$f(A,B,C,D)=\left\{\begin{matrix}\overline{A}\ \overline{B}\ \overline{D}+\overline{A}B\overline{C}+A\overline{B}C+ABD \\ \textbf{or} \\ \overline{A}\ \overline{C}\ \overline{D}+\overline{B}C\overline{D}+B\overline{C}D+ACD\end{matrix}\right\}$$

Larger Switching List Tabular Reductions

Of course, so far, all of the examples have had **four-variable** switching lists. If that was all that we were going to solve, a **Quine-McCluskey Tabular Reduction** solution would be overkill. These kinds of problems can easily be solved by **K-maps**; in much less time. However, in life, 4-variable expressions are not the norm. Systems usually have many more variables than that.

I don't teach **6-variable K-maps** although they are often used by engineers to solve problems. However, a **Q-M solution** would be very helpful if the engineer doesn't feel comfortable with **6-variable K-maps**. Above the **6-variable** level however, a **K-map** solution is not possible without using additional techniques taught in other courses (420). These types of problems are where **Tabular Reduction methods** shine!

Of course, another huge advantage of **Tabular Reduction** methods is that they lend themselves easily to **computerization**. At some point, the engineer stops doing these things by hand and turns them over to a computer program to solve. The only negative to this is that the more variables there are the more memory intensive the computer solution is.

The example has a **5-variable** switching list. We could go higher, but they would be very PAPER and TIME intensive. It is already an extremely long problem as it is. The point can be made by a **5-variable** list.

Quine-McCluskey Example 5 (5-Variable SOP):

Problem Statement: **Find the minimal SOP expression for the switching function below using QM Tabular Reduction**

You've seen this same problem in the **K-map** section.

$$f(A,B,C,D,E) = \Sigma m\left(\underbrace{2}_{1},\underbrace{3}_{2},\underbrace{7}_{3},\underbrace{8}_{1},\underbrace{9}_{2},\underbrace{10}_{2},\underbrace{11}_{3},\underbrace{12}_{2},\underbrace{13}_{3},\underbrace{15}_{4},\underbrace{21}_{3},\underbrace{24}_{2},\underbrace{25}_{3},\underbrace{26}_{3},\underbrace{27}_{4},\underbrace{28}_{3}\right)$$

Since this is our 1st **time** in the **5-variable Tabular Reduction world**, let's take this a bit more step-by-step then we have done in the last couple of problems. There are a couple of landmines which I have implanted in the problem to make it interesting and to make some educational points.

The figure to the right has the **min-term** list split up into **"# of ones"** order as always.

Set 1	**2** **8**	
Set 2	**3** **9** **10** **12** **24**	
Set 3	**7** **11** **13** **21** **25** **26** **28**	
Set 4	**15** **27**	

The "Group-of-Twos" Column

- **The groupings between the sets 1 and 2 are obvious (with one minor exception):**
  - 2 and 3 differ by (1) [a power of 2] ➔ 2,3(1)
    - 2 and 9 differ by (7) [FAIL]
    - 2 and 10 differ by (8) [a power of 2] ➔ 2,10(8)
    - 2 and 12 differ by (10) [FAIL]
    - 2 and 24 differ by (22) [FAIL]
  - 8 and 3 ➔ (8 > 3 so they can't be compared)🡐 🡐🡐🡐🡐🡐
    - 8 and 9 differ by (1) [a power of 2] ➔ 8,9(1)
    - 8 and 10 differ by (2) [a power of 2] ➔ 8,10(2)
    - 8 and 12 differ by (4) [a power of 2] ➔ 8,12(4)
    - 8 and 24 differ by (16) [a power of 2] ➔ 8,24(16)
- ***All min-terms which were used in creating these groups of two have been checked off.***

Set 1	**2 √** **8 √**	**2,3(1)** **2,10(8)** **8,9(1)** **8,10(2)** **8,12(4)** **8,24(16)**
Set 2	**3 √** **9 √** **10 √** **12 √** **24 √**	
Set 3	**7** **11** **13** **21** **25** **26** **28**	
Set 4	**15** **27**	

Special note should be taken of the MINOR EXCEPTION above:

- Look at the 8 and 3 comparison above. We have seen this situation once before in **Example 1**. Here it is again. This is another reminder that the **upper number MUST be smaller than the lower number.**

Example Continued on Next Page)

Example Continued)

- Moving on to comparing min-term **set 2** with **set 3**, we get:
    - 3 and 7 differ by (4) [a power of 2] → 3,7(4)
        - 3 and 11 differ by (8) [a power of 2] → 3,11(8)
        - 3 and 13 differ by (10) [FAIL]
        - 3 with 21, 25, 26, and 28 [FAIL]
    - 9 and 7 → (9 > 7 so they can't be compared)← ←←←←←
        - 9 and 11 differ by (2) [a power of 2] → 9,11(2)
        - 9 and 13 differ by (4) [a power of 2] → 9,13(4)
        - 9 and 21 differ by (12) [FAIL]
        - 9 and 25 differ by (16) [a power of 2] → 9,25(16)
        - 9 with 26 and 28 [FAIL]
        - 9 and 28 differ by (18) [FAIL]
    - 10 and 7 → (10 > 7 so they can't be compared)← ←←←←←
        - 10 and 11 differ by (1) [a power of 2] → 10,11(1)
        - 10 with 13, 21 and 25 [FAIL]
        - 10 and 26 differ by (16) [a power of 2] → 10,26(16)
        - 10 and 28 differ by (18) [FAIL]
    - 12 and 7 → (12 > 7 and 11 so they can't be compared)← ←←←←←
        - 12 and 13 differ by (1) [a power of 2] → 12,13(1)
        - 12 with 21, 25, and 26 [FAIL]
        - 12 and 28 differ by (16) [a power of 2] → 12,28(16)
    - 24 with 7, 13, and 21 →
        - (24 > {7, 11, 13 and 21} so they can't be compared)←
        - 24 and 25 differ by (1) [a power of 2] → 24,25(1)
        - 24 and 26 differ by (2) [a power of 2] → 24,26(2)
        - 24 and 28 differ by (4) [a power of 2] → 24,28(4)

Set 1	2 √ 8 √	2,3(1) 2,10(8) 8,9(1) 8,10(2) 8,12(4) 8,24(16)
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) 3,11(8) 9,11(2) 9,13(4) 9,25(16) 10,11(1) 10,26(16) 12,13(1) 12,28(16) 24,25(1) 24,26(2) 24,28(4)
Set 3	7 √ 11 √ 13 √ 21 25 √ 26 √ 28 √	
Set 4	15 27	

WOW! That was fun, wasn't it? It seemed to just keep on going. A couple of things to note about the process:

- We had several spots where the **top # was larger than the bottom one.**
- **Min-term 21 is presently missing a check-mark.** Perhaps that will be taken care of on the next page?

Example Continued on Next Page)

Example Continued)

Ok, now it's time to compare min-term set 3 with set 4. Perhaps this will be shorter!

- 7 and 15 differ by (8) [a power of 2] ➔ 7,15(8)
    - 7 and 27 differ by (18) [FAIL]
- 11 and 15 differ by (4) [a power of 2] ➔ 11,15(4)
    - 11 and 27 differ by (16) [a power of 2] ➔ 11,27(16)
- 13 and 15 differ by (2) [a power of 2] ➔ 13,15(2)
    - 13 and 27 differ by (14) [FAIL]
- 21 and 15 ➔ (21 > 15 so they can't be compared)←
    - 21 and 27 differ by (6) [FAIL]
- 25 and 15 ➔ (25 > 15 so they can't be compared)←
    - 25 and 27 differ by (2) [a power of 2] ➔ 25,27(2)
- 26 and 15 ➔ (26 > 15 so they can't be compared)←
    - 26 and 27 differ by (1) [a power of 2] ➔ 26,27(1)
- 28 and 15 ➔ (28 > 15 so they can't be compared)←
    - 28 and 27 ➔ (28 > 27 so they can't be compared)←

Set 1	2 √ 8 √	2,3(1) 2,10(8) 8,9(1) 8,10(2) 8,12(4) 8,24(16)
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) 3,11(8) 9,11(2) 9,13(4) 9,25(16) 10,11(1) 10,26(16) 12,13(1) 12,28(16) 24,25(1) 24,26(2) 24,28(4)
Set 3	7 √ 11 √ 13 √ 21 PI 25 √ 26 √ 28 √	7,15(8) 11,15(4) 11,27(16) 13,15(2) 25,27(2) 26,27(1)
Set 4	15 √ 27 √	X

- One new item in this one. Note that **min-term 21 never was used to make up a pair, therefore it got circled**. In all other columns circled items are **PI**'s. In this column, it is an **EPI** since it is analogous to a single **1** in a **K-map** **grouped by itself**. However, just to keep from confusing the issue at this point, we will mark it down as a **PI**.

Next we are going to work on the "**group of 4's**" column on the next page.

Example Continued on Next Page)

Example Continued)

The "**Group-of-Fours**" column

- Remember that in this column the pairs can't be compared with each other **unless they share the same #'s in ( ).**
- I suggest that you use the 1st # of each pair for the **comparison** and the 2nd # of each pair as a **double check**.

We will start by comparing **Pairs set 1** with **set 2**.

- Group 2,3(1) and 10,11(1)
  - 2 and 10 differ by (8) ➔ 2,3,10,11(1,8)
- Group 2,3(1) and 12,13(1)
  - 2 and 12 differ by (10) [FAIL]
- Group 2,3(1) and 24,25(1)
  - 2 and 24 differ by (22) [FAIL]
- Group 8,9(1) and 10,11(1)
  - 8 and 10 differ by (2) ➔ 8,9,10,11(1,2)
- Group 8,9(1) and 12,13(1)
  - 8 and 12 differ by (4) ➔ 8,9,12,13(1,4)
- Group 8,9(1) and 24,25(1)
  - 8 and 24 differ by (16) ➔ 8,9,24,25(1,16)
- Group 8,10(2) and 9,11(2)
  - 8 and 9 differ by (1) ➔ 8,10,9,11(2,1)
- Group 8,10(2) and 24,26(2)
  - 8 and 24 differ by (16) ➔ 8,10,24,26(2,16)
- Group 8,12(4) and 3,7(4)
  - 8 > 3 so they can't be compared. FAILED
- Group 8,12(4) and 9,13(4)
  - 8 and 9 differ by (1) ➔ 8,12,9,13(4,1)
- Group 8,12(4) and 24,28(4)
  - 8 and 24 differ by (16) ➔ 8,12,24,28(4,16)
- Group 2,10(8) and 3,11(8)
  - 2 and 3 differ by (1) ➔ 2,10,3,11(8,1)
- Group 8,24(16) and 9,25(16)
  - 8 and 9 differ by (1) ➔ 8,24,9,25(16,1)
- Group 8,24(16) and 10,26(16)
  - 8 and 10 differ by (2) ➔ 8,24,10,26(16,2)
- Group 8,24(16) and 12,28(16)
  - 8 and 12 differ by (4) ➔ 8,24,12,28(16,4)

Set 1	2 √ 8 √	2,3(1) √ 2,10(8) √ 8,9(1) √ 8,10(2) √ 8,12(4) √ 8,24(16) √	2,3,10,11(1,8) 8,9,10,11(1,2) 8,9,12,13(1,4) 8,9,24,25(1,16) 8,10,9,11(2,1) 8,10,24,26(2,16) 8,12,9,13(4,1) 8,12,24,28(4,16) 2,10,3,11(8,1) 8,24,9,25(16,1) 8,24,10,26(16,2) 8,24,12,28(16,4)
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) 3,11(8) √ 9,11(2) √ 9,13(4) √ 9,25(16) √ 10,11(1) √ 10,26(16) √ 12,13(1) √ 12,28(16) √ 24,25(1) √ 24,26(2) √ 24,28(4) √	
Set 3	7 √ 11 √ 13 √ 21 PI 25 √ 26 √ 28 √	7,15(8) 11,15(4) 11,27(16) 13,15(2) 25,27(2) 26,27(1)	
Set 4	15 √ 27 √	X	

All of the pairs which were used to make larger groups are checked-off. The group which has not been checked off might still get used with the next set so it is left alone.

Example Continued)

I will continue with my plan of taking the powers of two in order as I choose what groups to compare next.
Working with Pairs **sets** 2 and 3, we get:

Set 1	2 √ 8 √	2, 3(1) √ 2, 10(8) √ 8, 9(1) √ 8, 10(2) √ 8, 12(4) √ 8, 24(16) √	2, 3, 10, 11(1, 8) 8, 9, 10, 11(1, 2) 8, 9, 12, 13(1, 4) 8, 9, 24, 25(1, 16) 8, 10, 9, 11(2, 1) 8, 10, 24, 26(2, 16) 8, 12, 9, 13(4, 1) 8, 12, 24, 28(4, 16) 2, 10, 3, 11(8, 1) 8, 24, 9, 25(16, 1) 8, 24, 10, 26(16, 2) 8, 24, 12, 28(16, 4)
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3, 7(4) √ 3, 11(8) √ 9, 11(2) √ 9, 13(4) √ 9, 25(16) √ 10, 11(1) √ 10, 26(16) √ 12, 13(1) √ 12, 28(16) √ 24, 25(1) √ 24, 26(2) √ 24, 28(4) √	10, 11, 26, 27(1, 16) 24, 25, 26, 27(1, 2) 9, 11, 13, 15(2, 4) 9, 11, 25, 27(2, 16) 24, 26, 25, 27(2, 1) 3, 7, 11, 15(4, 8) 9, 13, 11, 15(4, 2) 3, 11, 7, 15(8, 4) 9, 25, 11, 27(16, 2) 10, 26, 11, 27(16, 1)
Set 3	7 √ 11 √ 13 √ $\boxed{21}$ PI 25 √ 26 √ 28 √	7, 15(8) √ 11, 15(4) √ 11, 27(16) √ 13, 15(2) √ 25, 27(2) √ 26, 27(1) √	X
Set 4	15 √ 27 √	X	X

Starting with the **(1)**'s:

- Group 10,11(1) and 26,27(1)
    - o 10 and 26 differ by (16) ➔ 10,11,26,27(1,16)
- Group 12,13(1) and 26,27(1)
    - o 12 and 26 differ by (14) ➔ FAILED
- Group 24,25(1) and 26,27(1)
    - o 24 and 26 differ by (2) ➔ 24,25,26,27(1,2)
- Group 9,11(2) and 13,15(2)
    - o 9 and 13 differ by (4) ➔ 9,11,13,15(2,4)
- Group 9,11(2) and 25,27(2)
    - o 9 and 25 differ by (16) ➔ 9,11,25,27(2,16)
- Group 24,26(2) and 13,15(2)
    - o 24 > 13 so they ➔ FAILED
- Group 24,26(2) and 25,27(2)
    - o 24 and 25 differ by (1) ➔ 24,26,25,27(2,1)
- Group 3,7(4) and 11,15(4)
    - o 3 and 11 differ by (8) ➔ 3,7,11,15(4,8)
- Group 9,13(4) and 11,15(4)
    - o 9 and 11 differ by (2) ➔ 9,13,11,15(4,2)
- Group 24,28(4) and 11,15(4)
    - o 24 > 11 so they ➔ FAILED
- Group 3,11(8) and 7,15(8)
    - o 3 and 7 differ by (4) ➔ 3,11,7,15(8,4)
- Group 9,25(16) and 11,27(16)
    - o 9 and 11 differ by (2) ➔ 9,25,11,27(16,2)
- Group 10,26(16) and 11,27(16)
    - o 10 and 11 differ by (1) ➔ 10,26,11,27(16,1)
- Group 12,28(16) and 11,27(16)
    - o 12 > 11 so they ➔ FAILED

A quick scan down the column verifies that all pairs have check-marks so it is time to delete the "**Duplicate groups**". Remember that **in the "Group-of-Fours" column every group will have its duplicate**.

Example Continued on Next Page)

Example Continued)

Before moving on to the "**Group of Eight**" column, we still need to delete the duplicate groups in the current column. Remember that:

- In this column, **<u>every group will have its duplicate</u>**. If you find one that doesn't then **YOU HAVE MAD A MISTAKE!!!!!!!**
- It is also important to note that while it is not the case in this problem, all the duplicates do not have to be in the same set as their partners. They could be in other sets in the columns.
- I suggest that you delete the group where the numbers are not in order. This will help to prevent your making comparison errors.

Set 1	2 √ 8 √	2,3(1) √ 2,10(8) √ 8,9(1) √ 8,10(2) √ 8,12(4) √ 8,24(16) √	2,3,10,11(1,8) 8,9,10,11(1,2) 8,9,12,13(1,4) 8,9,24,25(1,16) ~~8,10,9,11(2,1)~~ 8,10,24,26(2,16) ~~8,12,9,13(4,1)~~ 8,12,24,28(4,16) ~~2,10,3,11(8,1)~~ ~~8,24,9,25(16,1)~~ ~~8,24,10,26(16,2)~~ ~~8,24,12,28(16,4)~~
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) √ 3,11(8) √ 9,11(2) √ 9,13(4) √ 9,25(16) √ 10,11(1) √ 10,26(16) √ 12,13(1) √ 12,28(16) √ 24,25(1) √ 24,26(2) √ 24,28(4) √	10,11,26,27(1,16) 24,25,26,27(1,2) 9,11,13,15(2,4) 9,11,25,27(2,16) ~~24,26,25,27(2,1)~~ 3,7,11,15(4,8) ~~9,13,11,15(4,2)~~ ~~3,11,7,15(8,4)~~ ~~9,25,11,27(16,2)~~ ~~10,26,11,27(16,1)~~
Set 3	7 √ 11 √ 13 √ $\boxed{21}$ PI 25 √ 26 √ 28 √	7,15(8) √ 11,15(4) √ 11,27(16) √ 13,15(2) √ 25,27(2) √ 26,27(1) √	X
Set 4	15 √ 27 √	X	X

Up until now, things have been pretty much as they have been in previous examples with a few surprises thrown in. Now we are going to throw an even **BIGGER surprise** into the mix:

Note that there are <u>2 adjacent sets</u> in the "**group-of-fours**" column. This means that you will at least attempt to have a 4th column; a "**group-of-eights**" column.

- The thing special about this new column (other than the fact that there are **8 in a group**) is that in this column; **there will be triplicates, not duplicates!!!**

Example Continued on Next Page)

- **Example Continued)**

When grouping in this column, the groups can only be compared if **they share the same # in the ( ).** This is why I asked you to delete the **duplicates with the #'s out of order**. This way, the **#**'s in the **( )** will be in the **same order** so it will be **easier to recognize and harder to make a mistake**.

Set 1	2 √ 8 √	2,3(1) √ 2,10(8) √ 8,9(1) √ 8,10(2) √ 8,12(4) √ 8,24(16) √	2,3,10,11(1,8) PI 8,9,10,11(1,2) √ 8,9,12,13(1,4) PI 8,9,24,25(1,16) √ ~~8,10,9,11(2,1)~~ 8,10,24,26(2,16) √ ~~8,12,9,13(4,1)~~ 8,12,24,28(4,16) PI ~~2,10,3,11(8,1)~~ ~~8,24,9,25(16,1)~~ ~~8,24,10,26(16,2)~~ ~~8,24,12,28(16,4)~~	
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) √ 3,11(8) √ 9,11(2) √ 9,13(4) √ 9,25(16) √ 10,11(1) √ 10,26(16) √ 12,13(1) √ 12,28(16) √ 24,25(1) √ 24,26(2) √ 24,28(4) √	10,11,26,27(1,16) √ 24,25,26,27(1,2) √ 9,11,13,15(2,4) PI 9,11,25,27(2,16) √ ~~24,26,25,27(2,1)~~ 3,7,11,15(4,8) PI ~~9,13,11,15(4,2)~~ ~~3,11,7,15(8,4)~~ ~~9,25,11,27(16,2)~~ ~~10,26,11,27(16,1)~~	
Set 3	7 √ 11 √ 13 √ 21 PI 25 √ 26 √ 28 √	7,15(8) √ 11,15(4) √ 11,27(16) √ 13,15(2) √ 25,27(2) √ 26,27(1) √		
Set 4	15 √ 27 √	X		

- The following groups **do not share ( )** in the **adjacent set**. They are designated as **PI**s:
  - 2,3,10,11(1,8) (TOP SET)
  - 8,9,12,13(1,4) (TOP SET)
  - 8,12,24,28(4,16) (TOP SET)
  - 9,11,13,15(2,4) (BOTTOM SET)
  - 3,7,11,15(4,8) (BOTTOM SET)

Example Continued)

It should be noted that it is possible for a "**group-of-four**" to have a mate in the **adjacent set** but not differ by a **power of 2**. It just did not happen in this example.

Set 1	2 √ 8 √	2,3(1) √ 2,10(8) √ 8,9(1) √ 8,10(2) √ 8,12(4) √ 8,24(16) √	2,3,10,11(1,8) PI 8,9,10,11(1,2) √ 8,9,12,13(1,4) PI 8,9,24,25(1,16) √ ~~8,10,9,11(2,1)~~ 8,10,24,26(2,16) √ ~~8,12,9,13(4,1)~~ 8,12,24,28(4,16) PI ~~2,10,3,11(8,1)~~ ~~8,24,9,25(16,1)~~ ~~8,24,10,26(16,2)~~ ~~8,24,12,28(16,4)~~	8,9,10,11,24,25,26,27(1,2,16) PI ~~8,9,24,25,10,11,26,27(1,16,2)~~ ~~8,10,24,26,9,11,25,27(2,16,1)~~
Set 2	3 √ 9 √ 10 √ 12 √ 24 √	3,7(4) √ 3,11(8) √ 9,11(2) √ 9,13(4) √ 9,25(16) √ 10,11(1) √ 10,26(16) √ 12,13(1) √ 12,28(16) √ 24,25(1) √ 24,26(2) √ 24,28(4) √	10,11,26,27(1,16) √ 24,25,26,27(1,2) √ 9,11,13,15(2,4) PI 9,11,25,27(2,16) √ ~~24,26,25,27(2,1)~~ 3,7,11,15(4,8) PI ~~9,13,11,15(4,2)~~ ~~3,11,7,15(8,4)~~ ~~9,25,11,27(16,2)~~ ~~10,26,11,27(16,1)~~	X
Set 3	7 √ 11 √ 13 √ 21 PI 25 √ 26 √ 28 √	7,15(8) √ 11,15(4) √ 11,27(16) √ 13,15(2) √ 25,27(2) √ 26,27(1) √	X	X
Set 4	15 √ 27 √	X	X	X

- Group 8,9,10,11 (1,2) and Group 24,25,26,27 (1,2)
    - 8 differs from 24 by (16) ➔ 8,9,10,11,24,25,26,27 (1,2,16).
- Group 8,9,24,25(1,16) and Group 10,11,26,27(1,16)
    - 8 differs from 10 by (2) ➔ 8,9,24,25,10,11,26,27(1,16,2)
- Group 8,10,24,26(2,16) and Group 9,11,25,27(2,16)
    - 8 differs from 9 by (1) ➔ 8,10,24,26, 9,11,25,27(2,16,1)

The final step in this process it to delete the **unneeded triplicates**.

- The **bottom two are lined out** and the top one is circled and designated as a **PI**.

Example Continues)

The next to last step is to create the **PI** Table to determine the contents of the solution.

- List the **PI**'s along the left side and the **min-terms** along the top.

Prime Implicant Table (PI TABLE)

			2	3	7	8	9	10	11	12	13	15	21	24	25	26	27	28
EPI	21												☒					
EPI	8, 9, 10, 11, 24, 25, 26, 27	(1, 2, 16)				X	X	X	X					X	☒	☒	☒	
EPI	2, 3, 10, 11	(1, 8)	☒	X				X	X									
	8, 9, 12, 13	(1, 4)				X	X			X	X							
EPI	8, 12, 24, 28	(4, 16)				X				X				X				☒
	9, 11, 13, 15	(2, 4)					X		X		X	X						
EPI	3, 7, 11, 15	(4, 8)		X	☒				X			X						

- Once the table is created, perform and '**inventory**' by placing an **X** in the column of each **min-term** covered by a **PI**.

- Once this is done, look for '**LONELY Xs'** (Single **X**'s in a column)

Prime Implicant Table (PI TABLE)

			√	√	√	√	√	√	√	√		√	√	√	√	√	√	√
			2	3	7	8	9	10	11	12	$\boxed{13}$	15	21	24	25	26	27	28
EPI	21												☒					
EPI	8, 9, 10, 11, 24, 25, 26, 27	(1, 2, 16)				X	X	X	X					X	☒	☒	☒	
EPI	2, 3, 10, 11	(1, 8)	☒	X				X	X									
	8, 9, 12, 13	(1, 4)				X	X			X	X							
EPI	8, 12, 24, 28	(4, 16)				X				X				X				☒
PI	9, 11, 13, 15	(2, 4)					X		X		X	X						
EPI	3, 7, 11, 15	(4, 8)		X	☒				X			X						

- Once the **EPI**'s are designated; place a check-mark above any min-term which is covered by an **EPI**.
- If a min-term does not have a check-mark above it (not covered by an element which is essential the answer, **EPI**), then it has to be covered by
  - The smallest number of **PI**.s
  - Starting with the largest **PI**s

- In this case, the only min-term which was left uncovered is min-term 13. That min-term can only be covered by
  - 9,11,13,15. ➔ Designate that as a **PI** (thus part of the answer) to the left.

Example Continues on the next page)

Example Continues)

- We need to covert the **PI**s and **EPI**s to **Boolean variables**.
- Set the table up as shown below. I used the 1st number in each group for the **Binary Value**.

Prime Implicant Table (PI TABLE)

			16	8	4	2	1	
			A	B	C	D	E	Boolean
EPI	21		1	0	1	0	1	
EPI	8, 9, 10, 11, 24, 25, 26, 27	(1, 2, 16)	0	1	0	0	0	
EPI	2, 3, 10, 11	(1, 8)	0	0	0	1	0	
EPI	8, 12, 24, 28	(4, 16)	0	1	0	0	0	
PI	9, 11, 13, 15	(2, 4)	0	1	0	0	1	
EPI	3, 7, 11, 15	(4, 8)	0	0	0	1	1	

- Replace each bit in a row which is **pointed to by a # in the ( )** with a **–** or an **X**.

- Turn the bits which are left into their equivalent **Boolean variables**.

Prime Implicant Table (PI TABLE)

			16	8	4	2	1	
			A	B	C	D	E	Boolean
EPI	21		1	0	1	0	1	$A\overline{B}C\overline{D}E$
EPI	8, 9, 10, 11, 24, 25, 26, 27	(1, 2, 16)	–	1	0	–	–	$B\overline{C}$
EPI	2, 3, 10, 11	(1, 8)	0	–	0	1	–	$\overline{A}\ \overline{C}\ D$
EPI	8, 12, 24, 28	(4, 16)	–	1	–	0	0	$B\ \overline{D}\ \overline{E}$
PI	9, 11, 13, 15	(2, 4)	0	1	–	–	1	$\overline{A}BE$
EPI	3, 7, 11, 15	(4, 8)	0	–	–	1	1	$\overline{A}DE$

$$\boxed{f(A,B,C,D,E) = A\overline{B}C\overline{D}E + \overline{A}\ \overline{C}\ D + B\ \overline{D}\ \overline{E} + \overline{A}DE + B\overline{C} + \overline{A}BE}$$

- Move the Boolean terms into the **solution**. Place the **EPI** terms 1st followed by the **PI**s.