Quine-McCluskey and the Max-Term (F)

Up until now we have only solved **min-term equations** with the **Quine-McCluskey Tabular Reduction**. We can also solve **max-term** expressions. The nice thing about this is nothing changes in the procedure from that of the SOP Quine-McCluskey until it is time to convert the PI's and EPI's into Boolean algebra terms including the fact that you **still group the Max-terms by the # of 1's they contain**.

Let's take a look at another example. Just as before, we have seen this expression **k-mapped** before:

Quine-McCluskey Max-term Example:

Problem Statement: Simplify the following **Max-term** list into its minimal **POS** expression.

$$f(A,B,C,D,E) = \prod M\left(\underset{0}{0},\underset{1}{2},\underset{2}{5},\underset{1}{8},\underset{2}{10},\underset{2}{12},\underset{1}{16},\underset{2}{18},\underset{4}{23},\underset{2}{24},\underset{3}{26},\underset{4}{30}\right)$$

By now you should be fairly conversant with the procedure so we will condense the solution of this problem to entire columns.

- The **# of 1s** has been performed on the provided list above.

0 1's	0	√	0,2(2) 0,8(8) 0,16(16)
1 1's	2 8 16	√ √ √	2,10(8) 2,18(16) 8,10(2) 8,12(4) 8,24(16) 16,18(2) 16,24(8)
2 1's	5 10 12 18 24	PI √ √ √ √	10,26(16) 18,26(8) 24,26(2)
3 1's	26	√	26,30(4)
4 1's	23 30	PI √	X

The "Groups-of-Two column is pretty self-explanatory for the student who has studied the previous notes and understood the examples. Max-terms 5 and 23 were determined to be PIs since they were not used to build any group-of-two.

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Example Continues)

0 1's	0 √	0,2(2) √ 0,8(8) √ 0,16(16) √	0,2,8,10(2,8) 0,2,16,18(2,16) ~~0,8,2,10(8,2)~~ 0,8,16,24(8,16) ~~0,16,2,18(16,2)~~ ~~0,16,8,24(16,8)~~
1 1's	2 √ 8 √ 16 √	2,10(8) √ 2,18(16) √ 8,10(2) √ 8,12(4) **PI** 8,24(16) √ 16,18(2) √ 16,24(8) √	8,10,24,26(2,16) 16,18,24,26(2,8) 2,10,18,26(8,16) ~~16,24,18,26(8,2)~~ ~~2,18,10,26(16,8)~~ ~~8,24,10,26(16,2)~~
2 1's	5 **PI** 10 √ 12 √ 18 √ 24 √	10,26(16) √ 18,26(8) √ 24,26(2) √	X
3 1's	26 √	26,30(4) **PI**	X
4 1's	23 **PI** 30 √	X	X

The "**Group-of-Four**" column again demonstrates that **every term in that column has a duplicate**. The duplicates **HAVE** to be lined out **(NOT DELETED OR ERASED)** before moving on.

Example Continues on the next page)

Example Continues)

0 1's	0 √	0,2(2) √ 0,8(8) √ 0,16(16) √	0,2,8,10(2,8) √ 0,2,16,18(2,16) √ ~~0,8,2,10(8,2)~~ 0,8,16,24(8,16) √ ~~0,16,2,18(16,2)~~ ~~0,16,8,24(16,8)~~	0,2,8,10,16,18,24,26(2,8,16) PI ~~0,2,16,18,8,10,24,26(2,16,8)~~ ~~0,8,16,24,2,10,18,26(8,16,2)~~
1 1's	2 √ 8 √ 16 √	2,10(8) √ 2,18(16) √ 8,10(2) √ 8,12(4) PI 8,24(16) √ 16,18(2) √ 16,24(8) √	8,10,24,26(2,16) √ 16,18,24,26(2,8) √ 2,10,18,26(8,16) √ ~~16,24,18,26(8,2)~~ ~~2,18,10,26(16,8)~~ ~~8,24,10,26(16,2)~~	
2 1's	5 PI 10 √ 12 √ 18 √ 24 √	10,26(16) √ 18,26(8) √ 24,26(2) √	X	
3 1's	26 √	26,30(4) PI	X	
4 1's	23 PI 30 √	X	X	

Note that this time the table has a "Group of Eight" column.

- ***As you learned in the last set of notes, just as the "Group-of-Four" column will have 'duplicates' of each new term, the 'Group-of-Eight' column will have 'triplicates' of each new term. Two of each triplicate needs to be lined out. And as always, if a term does not have a complete set of triplicates, then YOU HAVE MAD A MISTAKE!***

Example Continues on the Next Page)

- The **PI table** is still performed the same way as it would be if the terms were **min-terms**.

PI TABLE

			√ 0	√ 2	√ 5	√ 8	√ 10	√ 12	√ 16	√ 18	√ 23	√ 24	√ 26	√ 30
EPI	5				[X]									
EPI	23										[X]			
EPI	8,12	(4)				X		[X]						
EPI	26,30	(4)											X	[X]
EPI	0,2,8,10,16,18,24,26	(2,8,16)	[X]	[X]		X	[X]		[X]	[X]		[X]	X	

- ***The Boolean Conversion table is the 1st place in the example where things are done differently than it would be done with min-terms.***

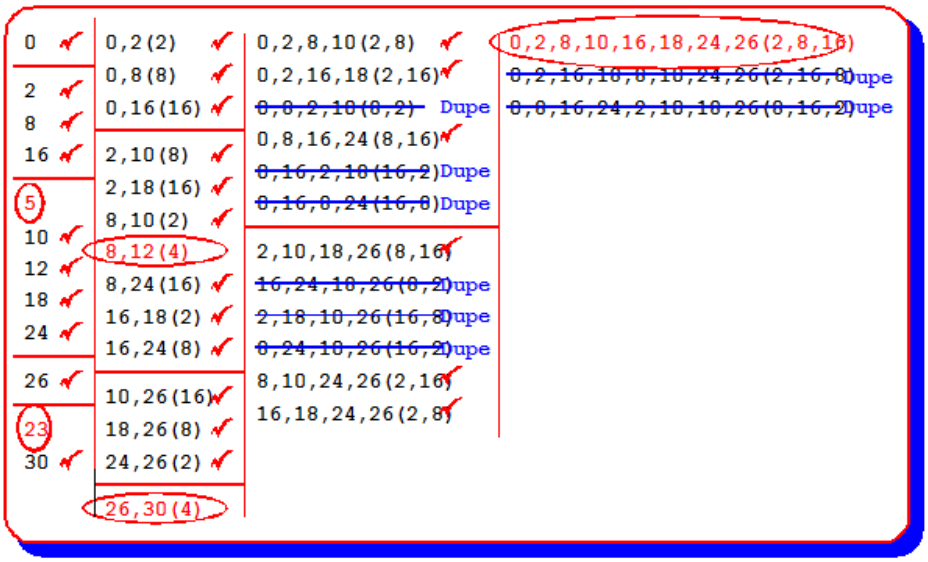
- ***Everything is the same until the terms are finally converted to Boolean. Then a 1 will give you a NOT'ed variable, while a 0 will give you a non-negated variable.***

- ***The resulting set of terms are then ANDed together into a POS expression.***

Boolean Conversion Table

		16 A	8 B	4 C	2 D	1 E	Boolean
EPI	5	0	0	1	0	1	$(A+B+\overline{C}+D+\overline{E})$
EPI	23	1	0	1	1	1	$(\overline{A}+B+\overline{C}+\overline{D}+\overline{E})$
EPI	8,12(4)	0	1	–	0	0	$(A+\overline{B}+D+E)$
EPI	26,30(4)	1	1	–	1	0	$(\overline{A}+\overline{B}+\overline{D}+E)$
EPI	0,2,8,10,16,18,24,26(2,8,16)	–	–	0	–	0	$(C+E)$

$$f(A,B,C,D,E)=(C+E)(A+B+\overline{C}+D+\overline{E})(\overline{A}+B+\overline{C}+\overline{D}+\overline{E})(A+\overline{B}+D+E)(\overline{A}+\overline{B}+\overline{D}+E)$$



Max – terms ⇒ / PI's ⇓	V 0	V 2	V 5	V 8	V 10	V 12	V 16	V 18	V 23	V 24	V 26	V 30
EPI ⇒ 0,2,8,10,16,18,24,26(2,8,16)	X	X		X	X		X	X		X	X	
EPI ⇒ 8,12(4)				X		X						
EPI ⇒ 26,30(4)											X	X
EPI ⇒ 5			X									
EPI ⇒ 23									X			

Selected PI's ⇓		16 A	8 B	4 C	2 D	1 E	
0,2,8,16,18,24,26	(2,8,16)	x	x	0	x	0	$C + E$
8,12	(4)	0	1	x	0	0	$A + \bar{B} + D + E$
26,30	(4)	1	1	x	1	0	$\bar{A} + \bar{B} + \bar{D} + E$
5		0	0	1	0	1	$A + B + \bar{C} + D + \bar{E}$
23		1	0	1	1	1	$\bar{A} + B + \bar{C} + \bar{D} + \bar{E}$

The minimized solution for the given function is:

$$f(A,B,C,D,E) = (C+E)(A+\bar{B}+D+E)(\bar{A}+\bar{B}+\bar{D}+E) \bullet (A+B+\bar{C}+D+\bar{E})(\bar{A}+B+\bar{C}+\bar{D}+\bar{E})$$