Multiple Output Quine-McCluskey (H)

Up until now, all the algebra, K-maps, and Quine-McCluskey techniques and examples which have been covered involved multiple input circuits with a SINGLE OUTPUT. Now we will see that one of the advantages that Quine-McCluskey has over K-maps is the ability to minimize circuits with MUTLIPLE outputs.

Multiple Output Quine-McCluskey Example:

Problem Statement: Minimize the following multiple output expression.

$$f_{\alpha}\left(A, B, C, D\right) = \sum m\left(\underbrace{4}_{1}, \underbrace{5}_{2}, \underbrace{6}_{2}, \underbrace{15}_{4}\right) + d\left(\underbrace{8}_{1}, \underbrace{11}_{3}\right)$$
$$f_{\beta}\left(A, B, C, D\right) = \sum m\left(\underbrace{0}_{0}, \underbrace{2}_{1}, \underbrace{3}_{2}, \underbrace{4}_{1}, \underbrace{5}_{2}\right) + d\left(\underbrace{8}_{1}, \underbrace{11}_{3}\right)$$

0	β√	0, 2(2) 0, 4(4) 0, 8(8)	β β β
2	β√	2, 3(1)	β
4	<mark>α.√</mark> β√	4,5(1)	αβ
8	α. β√	4, 6(2)	α
3	β√		
5	<mark>α√</mark> β√	3,11(8)	β
6	α√		
11	α√β√	11,15(4)	α
15	α√		

The min-terms (and don't cares if there are any) are listed in the 1st column in the same way as before. However, this time we need to keep track of which output the min-term (don't care) belongs to.

We can only compare terms with terms in the next group **down if they share the same output**. The resulting pair will only have the **output ID which is shared by both terms**.

Note that the check marks go beside output variable which was used.

Note here that min-term 8 was not used so it has been circled. Since it is in the 1st column it will be an **EPI**, not just a **PI** like the other columns.

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Example Conti	nued)												
		I		1	1.7	1	1	I	I	I	1	1	
				V	i ∨	V	V			i I	V	V	
	Min – terms 🍾			α	α	α	α	β	β	β	β	β	
	PI's U			4	5	6	15	0	2	3	4	5	
	0,2(2)		β		1	 	 	Х	Х	 	 	 	
	0, 4(4)	1	β	1	 	 !) 	X	1 !		X		
	0,8(8)	1	β	1				X	; !				
	2, 3(1)	1	β	1			 ! !		X	ΪXΤ			
$EPI \Rightarrow$	4,5(1)	α	β	Х	\mathbb{X}	 	 		+ 	 	Х	X	
$EPI \Rightarrow$	4,6(2)	α		Х]	\mathbf{X}					 		
	3, 1 1(8)		β							X	1	 	
EPI ⇒	11,15(4)	α					\mathbf{X}						
	×	α	β							 			

Note that there are three min-terms that need to be covered by **Beta PI's**. There are several different choices. Even though all the choices are all the same sizes, there might be some that are better choices than others. There are several ways to choose the best PI's. One way is called the "**Reduced PI table**".

Multiple Output Q-M (H)

3.29 The Reduced PI table:

As noted in the last paragraph, the EPI's were located but they don't cover all of the min-terms between them. Min-terms # 0, 2, and 3 on the Beta side are left uncovered and must be covered by a single or a combination of other PI's. We select which PI to use by creating a **Reduced PI table**. We remove the EPI's and the min-terms which are covered by them which just leaves the PI's in question and the uncovered minterms.

	Un cov ered		ß	ß	ß
	Min – terms 🍾		٢	Р	P
ROW	PI's Ų		0	2	3
1	0,2(2)	β	Х	Х	
2	0,4(4)	β	X		
3	0,8(8)	β	X		
4	2, 3(1)	β		Χ	Х
5	3,11(8)	β		 	Х

			A	в	C	D		COVERS			
epi	4,5	(1)	0	1	0	×	ABC	(both)			
epi	4,6	(2)	0	1	×	0	ABD	(a only)			
epi	11,15	(4)	1	×	1	1	ACD	(a only)			
pi	0,2	(2)	0	0	×	0	ĀBD	(β only)			
pi	2,3	(1)	0	0	1	×	ĀĒC	(β only)			
$f_{\alpha}(A, B, C, D) = \overline{A}B\overline{C} + \overline{A}B\overline{D} + ACD$											
$f_{\beta}(A, B, C, D) = \overline{A}\overline{B}\overline{C} + \overline{A} \overline{B} \overline{D} + \overline{A} \overline{B}C$											