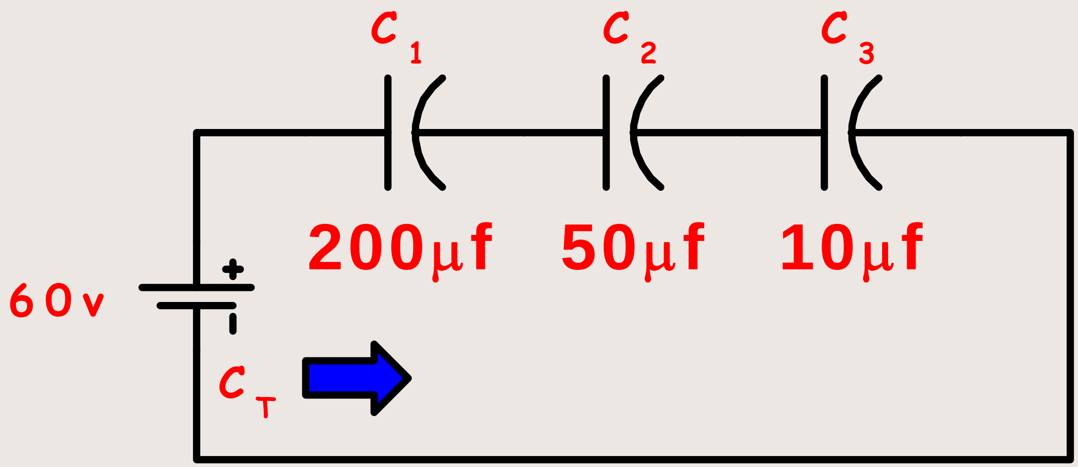
FE Review

RC and RL Analysis

Series Capacitors
Treat like parallel resistors

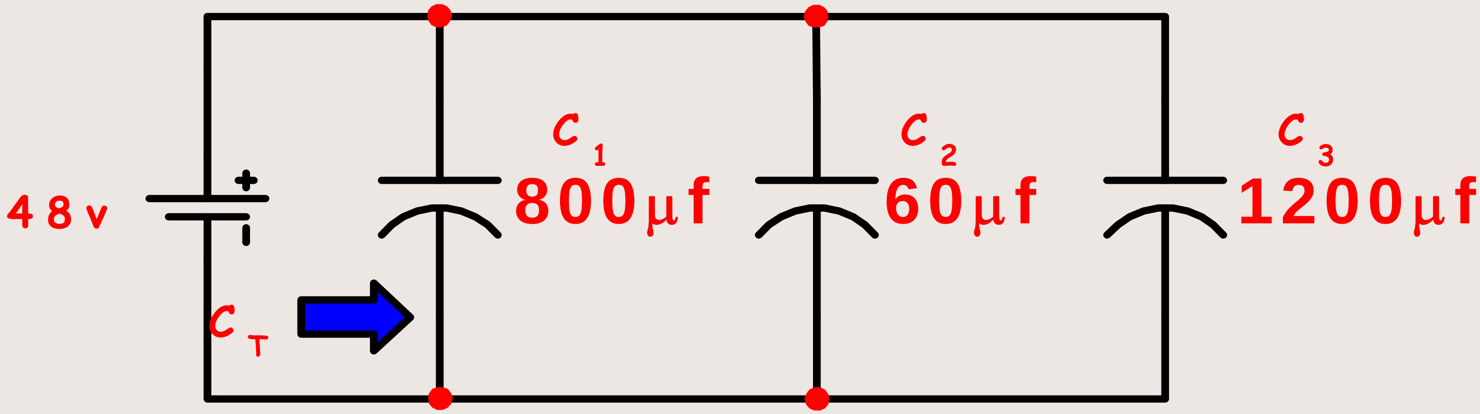


$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_T} = \frac{1}{200\mu f} + \frac{1}{50\mu f} + \frac{1}{10\mu f} = 125k$$

$$C_T = 8\mu f$$

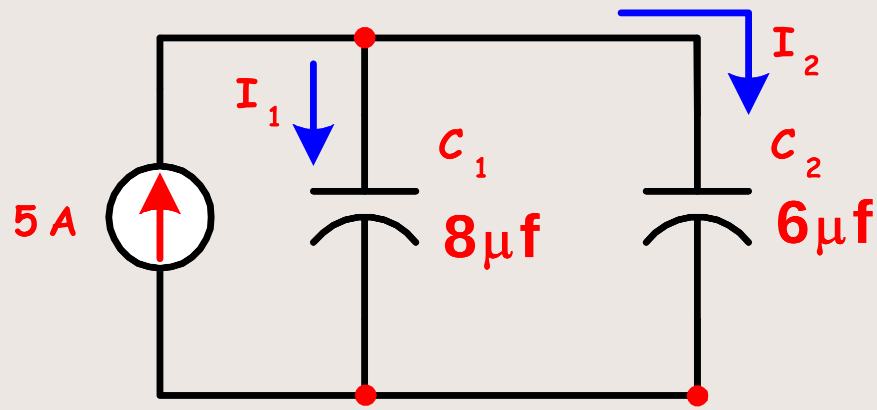
Parallel Capacitors
Treat like series resistors



$$C_T = C_1 + C_2 + C_3$$

$$= 800\mu f + 60\mu f + 1200\mu f = \boxed{2.06mf}$$

Capacitor Current Divider (Like a resistor voltage divider)

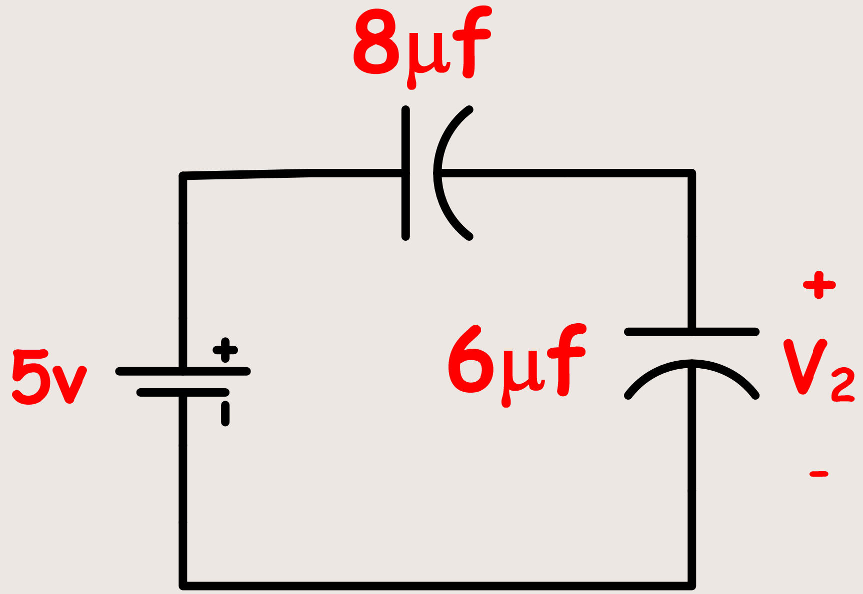


$$I_1 = \frac{IC_1}{C_1 + C_2} = \frac{5A(8\mu f)}{8\mu f + 6\mu f} = \boxed{2.857A}$$

$$I_2 = \frac{IC_2}{C_1 + C_2} = \frac{5A(6\mu f)}{8\mu f + 6\mu f} = \boxed{2.143A}$$

The two currents together MUST add up to 5 amps (Naturally) (taking roundoff error into account)

Capacitor voltage divider
(Like a resistor current divider)



$$V_2 = \frac{5vC_1}{C_1 + C_2} = \frac{5v\,(8\mu f)}{8\mu f + 6\mu f} = \frac{40v}{14} = \boxed{2.857v}$$

Inductors

Treat inductors just like resistors. This includes the inductive voltage and current divider equations.

First Order Fundamentals

Final Value
(steady state value)

$$v(t) = v(\infty) + \left[v(0^+) - v(\infty) \right] e^{-t/\tau}$$

Initial Value

time
constant

Initial and Final Values

To start with, let's consider a switch. In analyzing the switch, we will consider two extremes:

- ♦ The instant a switch is closed (the **Initial Value**), and
- ♦ the **steady state value** (a long time has passed since the switch closed) (The **Final Value**).

When you have a circuit in which you are working out the initial values,

- An **uncharged capacitor** acts as a **short circuit**. **(It opposes a change in voltage)**
- A **un-fluxed inductor** acts as an **open circuit**. **(It opposes a change in current)**.

Extending this idea to the device with an Initial charge, we get:

$$V_C(0+) = V_o \Leftarrow \text{a voltage source!}$$

$$i_L(0+) = I_o \Leftarrow \text{a current source!}$$

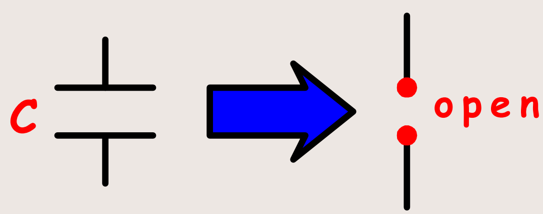
Final Conditions

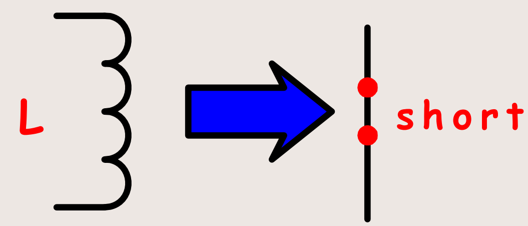
$i_c(t \Rightarrow \infty) = 0$ Capacitor current $\Rightarrow 0$ [open]

$v_L(t \Rightarrow \infty) = 0$ Inductor voltage $\Rightarrow 0$ [short]

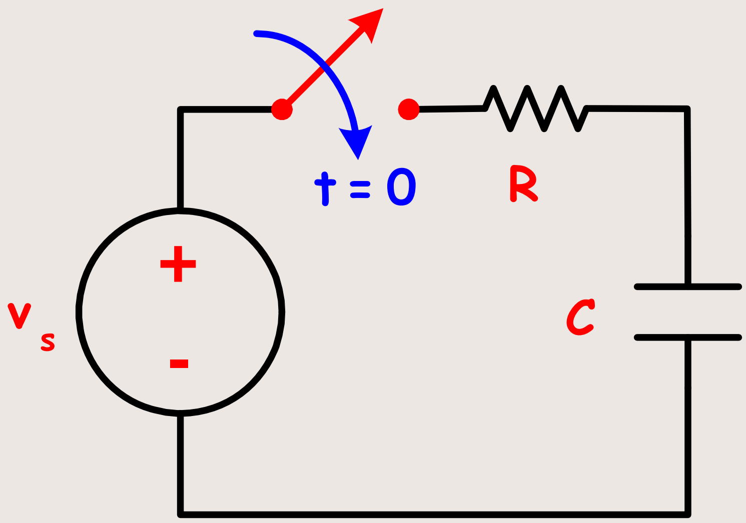
another way to write these values would be:

$i_c(\infty) = 0$ and $v_L(\infty) = 0$





RC circuit – Charge Phase



$$\tau = RC$$

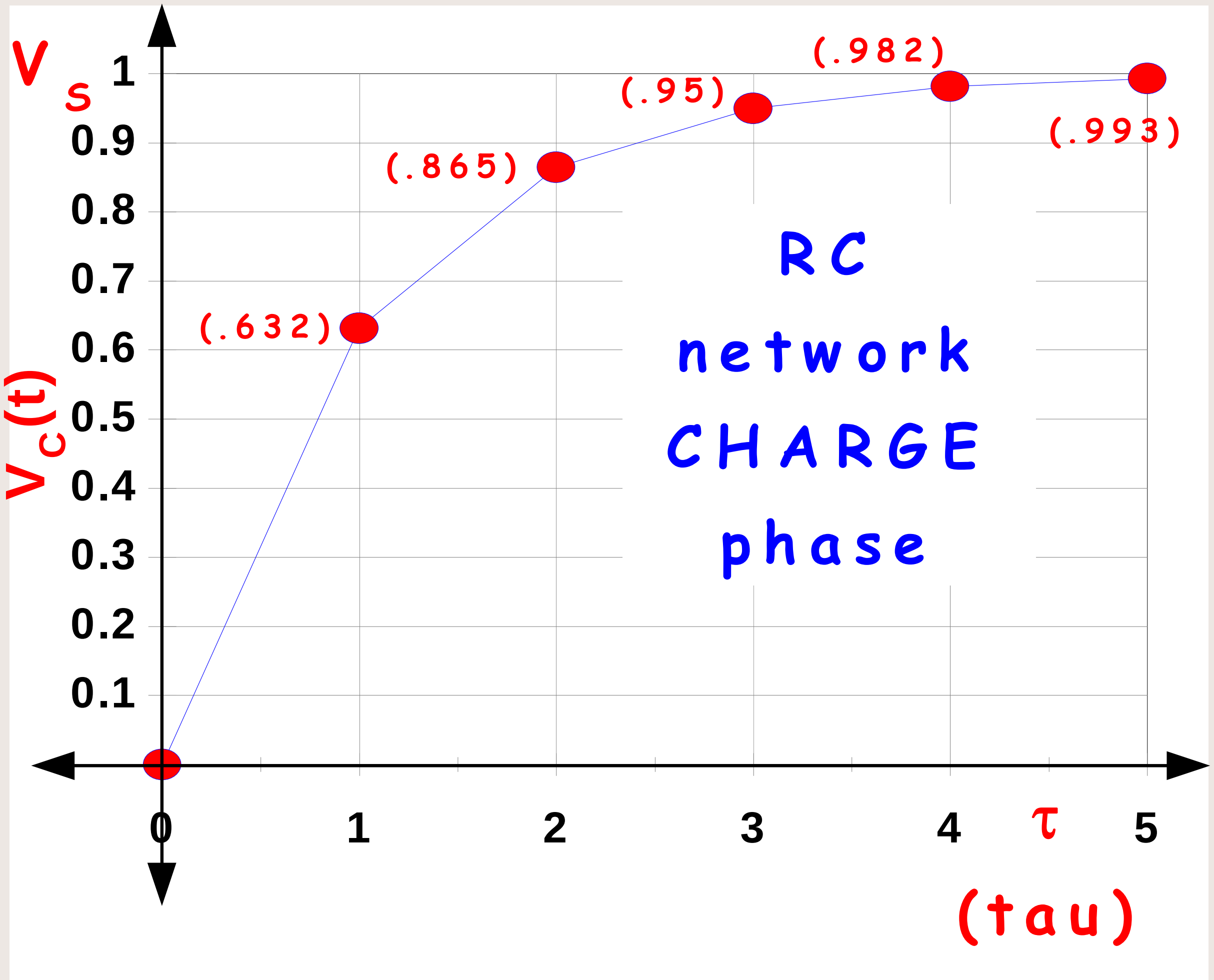
$$y = V_C(t)$$

$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right] e^{-t/\tau}$$

$$\left.\begin{aligned} y(0^+) &= V_C(0^+) = 0v \\ y(\infty) &= V_C(\infty) = V_S \end{aligned}\right\}$$ determined earlier

$$V_c(t) = V_S + [0v - V_S] e^{-t/RC}$$

$$\boxed{V_C(t) = V_S\left(1 - e^{-t/RC}\right)}$$



V
S
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Vc(t)
(.632)
(.865)
(.95)
(.982)
(.993)
RC
network
CHARGE
phase
0
1
2
3
4
τ
5
(tau)

RC Circuit - Discharge Phase

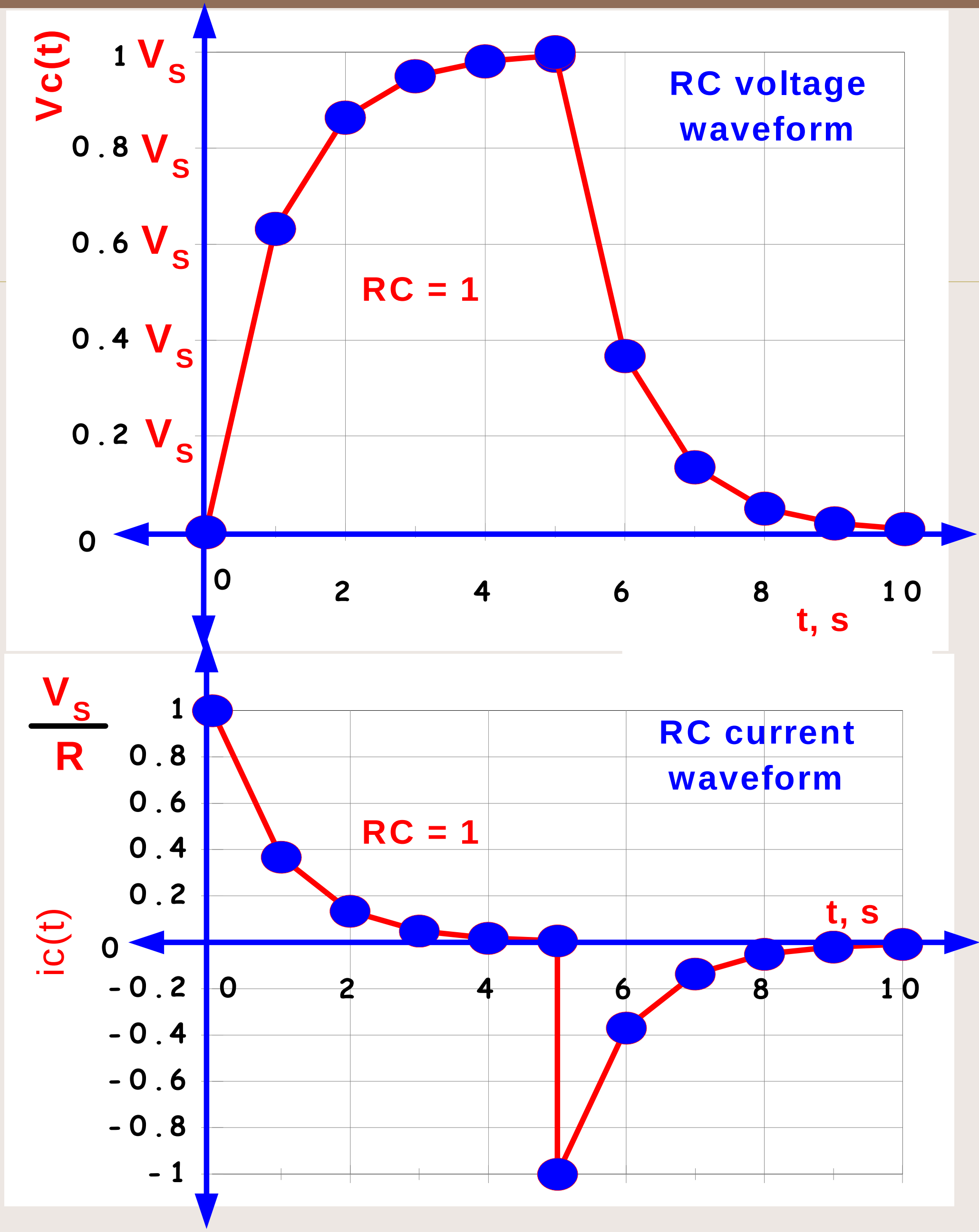
Discharge phase - Voltage

$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right] e^{-t/\tau}$$

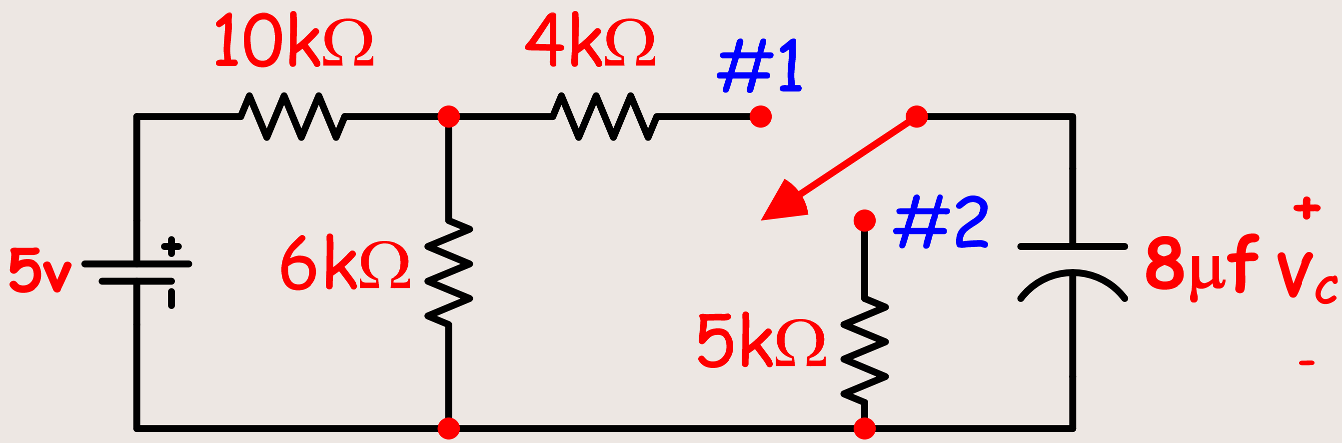
$V_C(0^+) = V_S \quad V_C(\infty) = 0v$

Note that it is assumed here that the capacitor had time to completely charge to V_s.

$$V_c(t) = 0v + \left[V_S - 0v\right] e^{-t/RC} \Rightarrow \boxed{V_C(t) = V_S e^{-t/RC}}$$

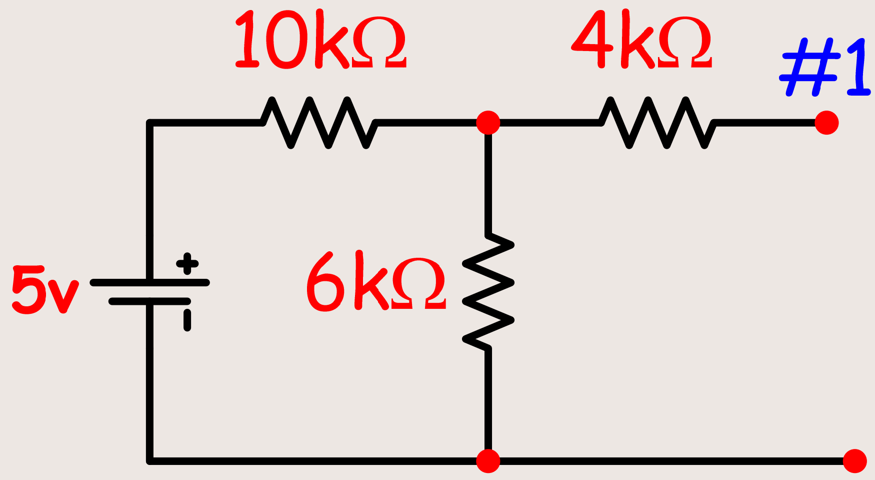

Vc(t)
1 Vs
0.8 Vs
0.6 Vs
0.4 Vs
0.2 Vs
0
RC voltage waveform
RC = 1
0
2
4
6
8
10
t, s
Vs / R
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
ic(t)
RC current waveform
RC = 1
t, s
0
2
4
6
8
10

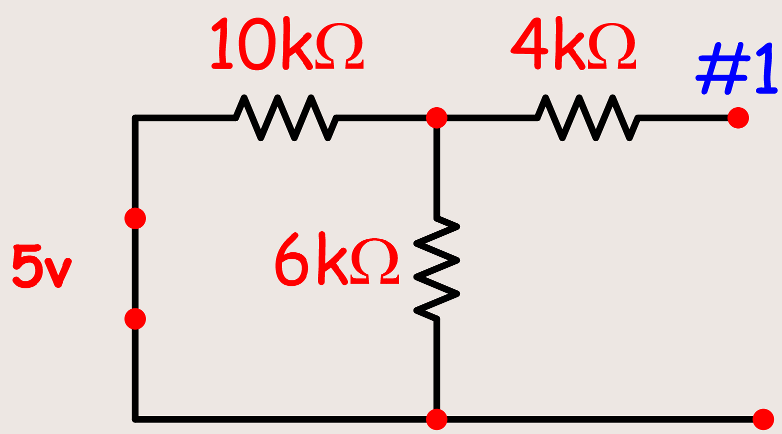
Example



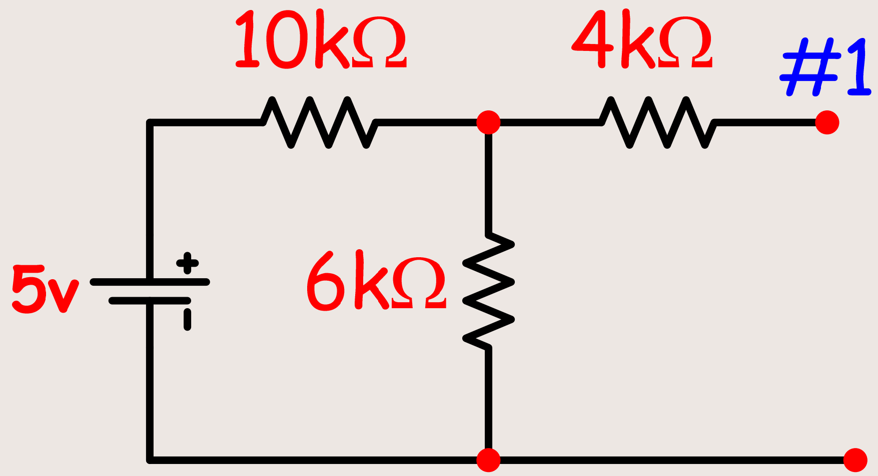
At time t = 0 the switch goes to position #1.
The initial charge on the capacitor is 0 volts.
Show the charge graph of the capacitor voltage.

Step 1: Remove the load (the capacitor) and the Discharge Resistor and determine the Thevenin Ckt.





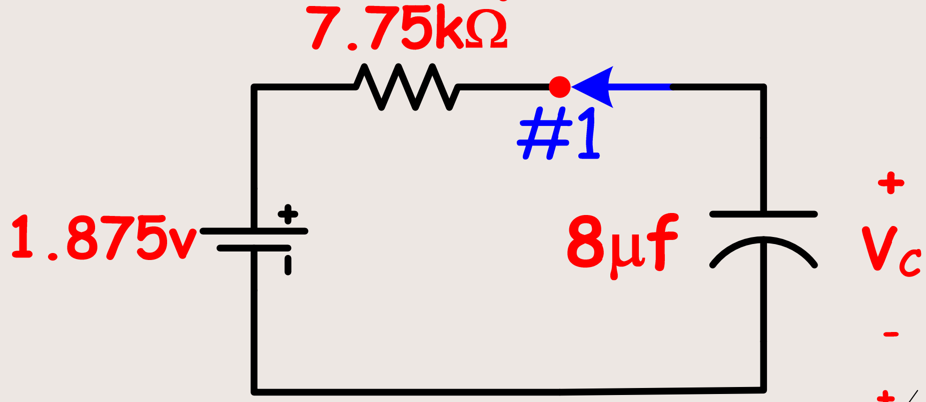
$$R_{Th} = 10k\Omega \,||\, 6k\Omega + 4k\Omega$$
$$= 3.75k\Omega + 4k\Omega$$
$$= \boxed{7.75k\Omega}$$



$$V_{Th} = \frac{5v\,(6k\Omega)}{10k\Omega + 6k\Omega}$$

$$= 1.875v$$

Place the Cap onto the Thevenin circuit and then calculate the charge eqn.



$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right]e^{-t/\tau}$$

$$\tau = RC = 7.75k\Omega(8\mu f) = 62ms$$

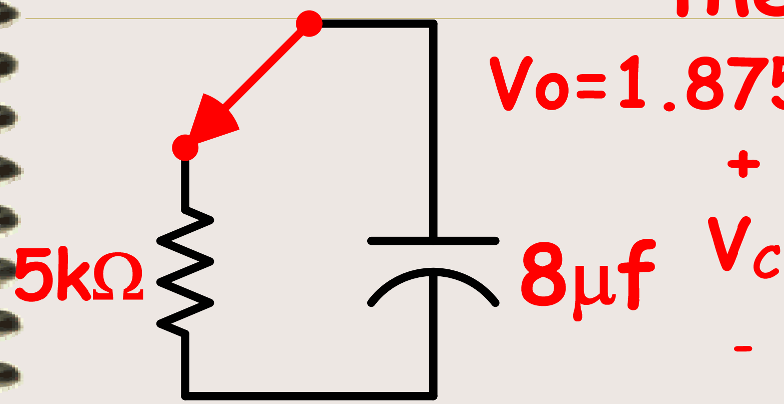
$$V_c(0^+) = 0v \quad V_c(\infty) = 1.875v$$

$$V_c(t) = 1.875v + [0v - 1.875v]e^{-t/62ms}$$

$$= 1.875 - 1.875e^{-t/62ms}$$

$$= 1.875\left(1 - e^{-t/62ms}\right)$$

Now place the cap with the discharge resistor and recalculate the eqn.



$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right]e^{-t/\tau}$$

$$\tau = RC = 5k\Omega(8\mu f) = 40ms$$

$$V_c(0^+) = 1.875v \quad V_c(\infty) = 0v$$

$$V_c(t) = 0v + [1.875v - 0v]e^{-t/\tau}$$

$$V_c(t) = 1.875ve^{-t/40ms}$$