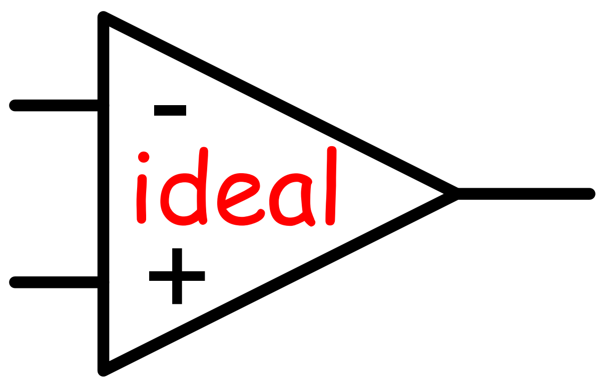
FE REVIEW

OPERATIONAL AMPLIFIERS (OP-AMPS)

The Op-amp

An op-amp has two inputs and one output. Note the op-amp below. The terminal labeled with the (-) sign is the "inverting" input and the input labeled with the (+) sign is the "non-inverting" terminal. The standard way to show the device is with the "inverting" terminal on top but this is not always the case. BE CAREFUL!!

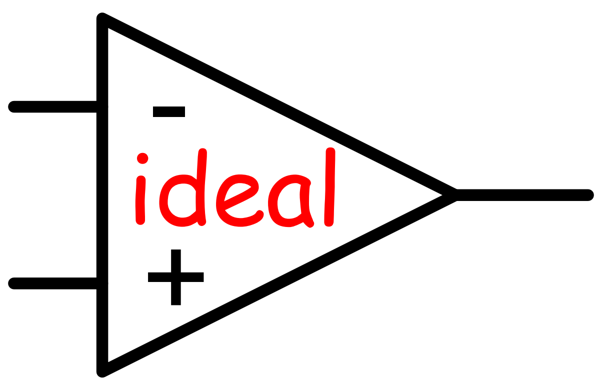


The "Ideal" assumptions

When we assume "Ideal" characteristics for an op-amp we are able to make several assumptions which make analysis MUCH easier.

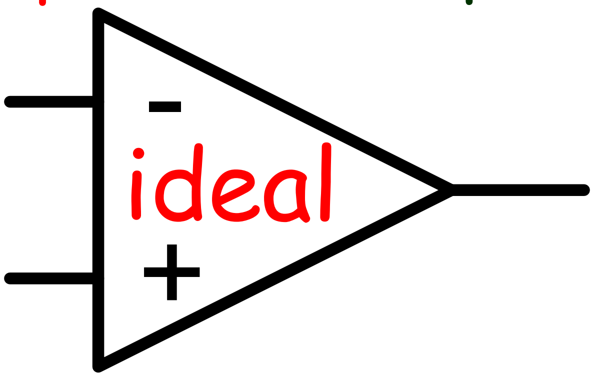
The first of these is that the input resistance of each of the two terminals is "Infinite".

$$R_{in} = \infty$$



“Ideal” assumptions continued)

The next major assumption is that the output impedance is equal to zero.



$$R_{out} = 0$$

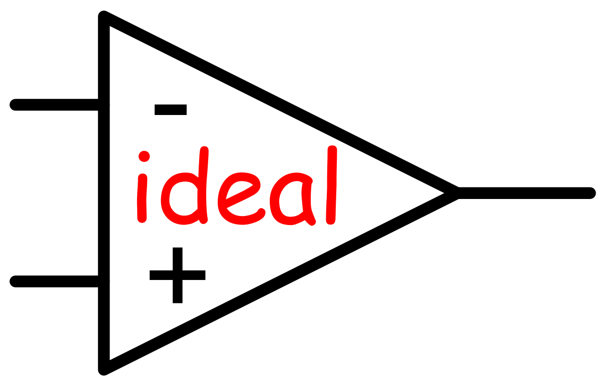
This assumption allows us to make the further assumption that V_{out} is independent of the load. This means that it can not be loaded down.

"Ideal" assumptions continued)

The next assumption is that the "open-loop" gain (A) is infinite (in real life if is in the millions so this is a safe assumption).

Open – Loop Gain

$A = \infty$



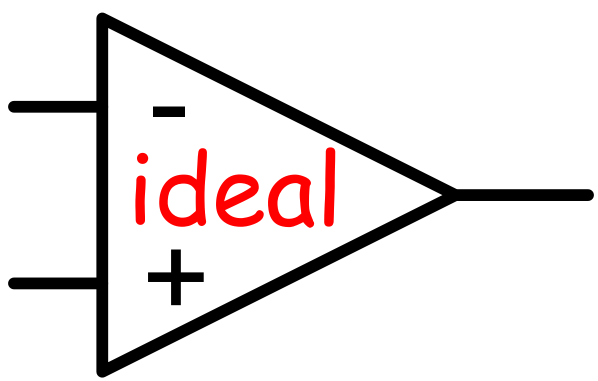
"Ideal" assumptions continued)

The assumption that *A* is infinite allows us to make a further assumption that the voltage on each terminal is equal to each other. This is concept is called a "Virtual Short". This assumption turns out to be the KEY to op-amp circuit analysis.

$$V_{out} = A(V^{+} - V^{-})$$
$$= AV_d$$

$$\therefore V_d = \frac{V_{out}}{A} \quad \text{so,} \quad \lim_{A \to \infty} \frac{V_{out}}{A} = 0$$

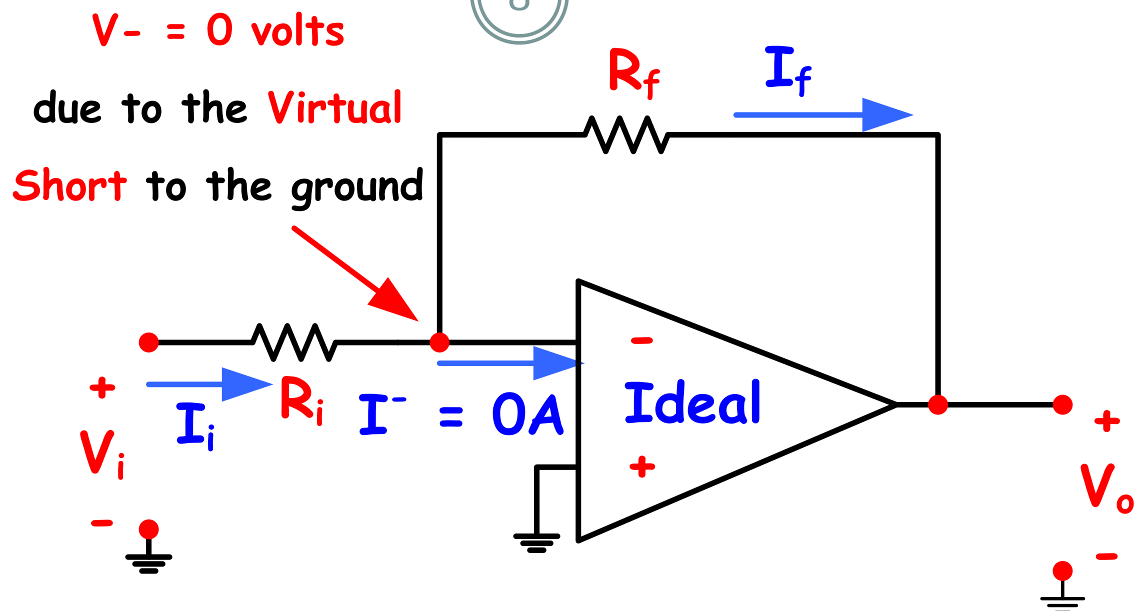
it follows that $V^{+} = V^{-}$



“Ideal” assumptions continued)

A “Virtual” short is not like a real short (or it wouldn’t be virtual). A virtual short only affects voltage, not current.

The Inverting Op-amp



With the addition of two resistors and a ground on the non-inverting terminal we get an "inverting" op-amp. Note that since the non-inverting terminal is grounded **the voltage on the "inverting" terminal is also grounded due to the "Virtual" short**.

The Inverting Op-amp continued)

Note that as long as there isn't a voltage source on the non-inverting terminal there could be a dozen resistors on it and it still would not affect the voltage on that terminal.

Since the current is assumed to be zero on the "inverting" terminal, KCL proves that current I_i is equal to the feedback current I_f.

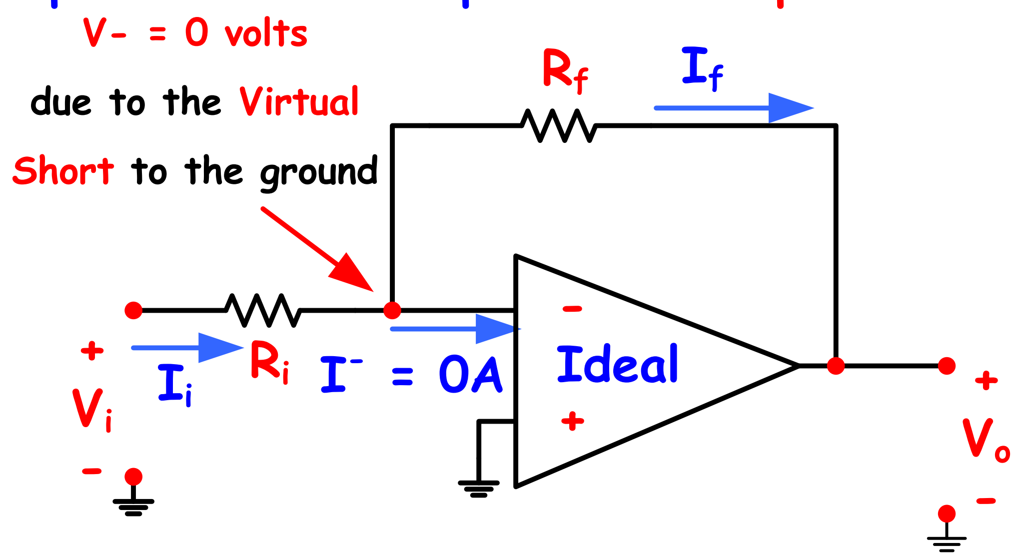
$$0 = -I_i + I_f + I^-$$

$$I_i = I_f + 0 \quad \Rightarrow \quad I_i = I_f$$

The Inverting Op-amp continued)

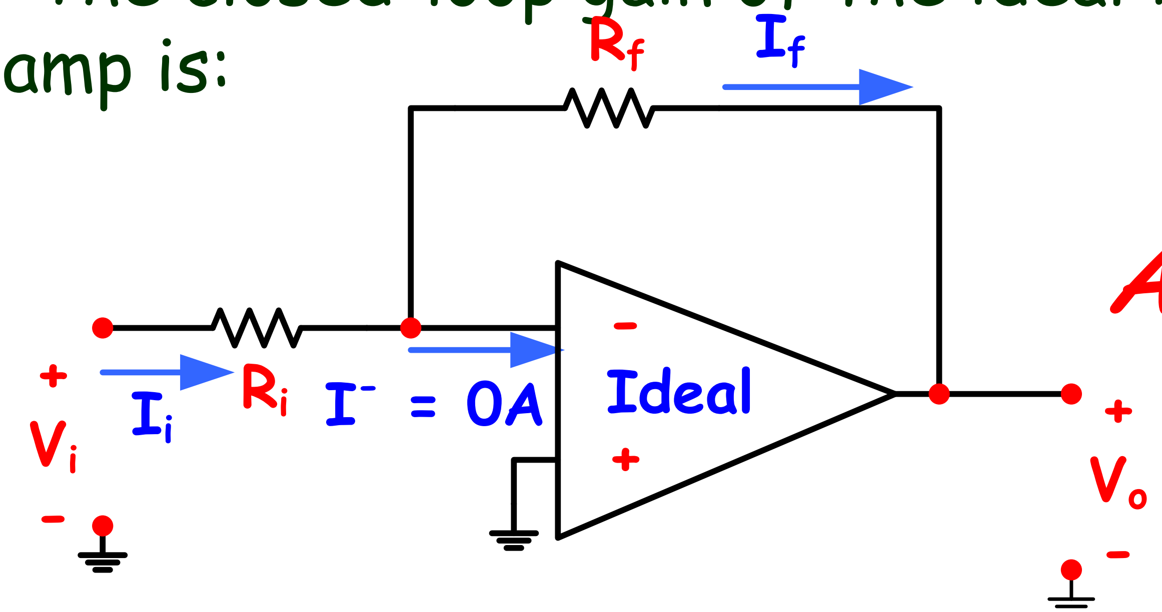
Another aspect of the virtual short is that the input resistance of the "inverting" op-amp is equal to R_i.

$$I_i = \frac{V_i - V^-}{R_i} = \frac{V_i - 0\ v}{R_i} = \frac{V_i}{R_i} \Leftarrow$$



The Inverting Op-amp continued)

The closed-loop gain of the ideal inverting op-amp is:

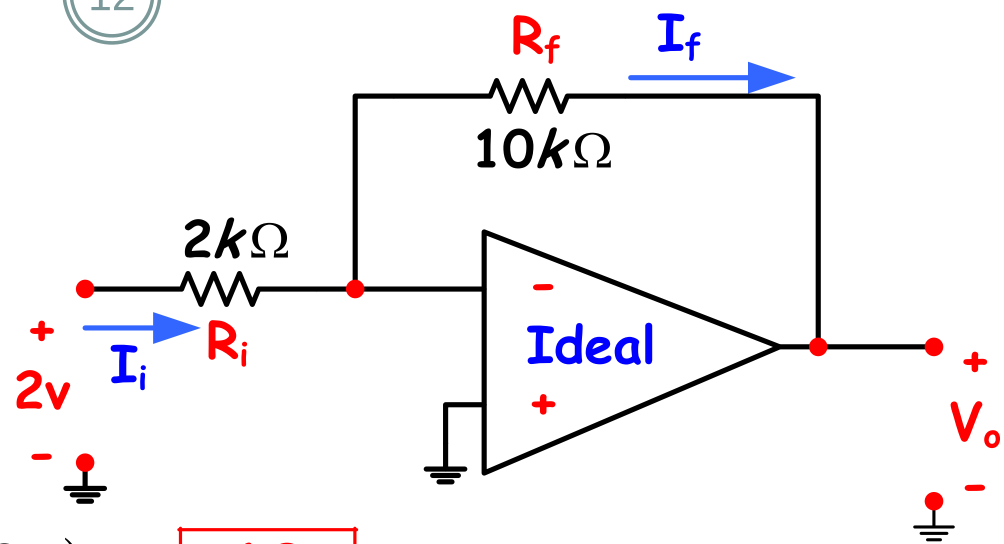


$$A_{CL} = \frac{-R_f}{R_i}$$

The FE exam uses a (A) for closed-loop gain.

The Inverting Op-amp continued)

Find V_o, I_i, I_f, and R_{in} for the circuit shown:



$$V_o = \frac{-R_f}{R_i} V_i$$

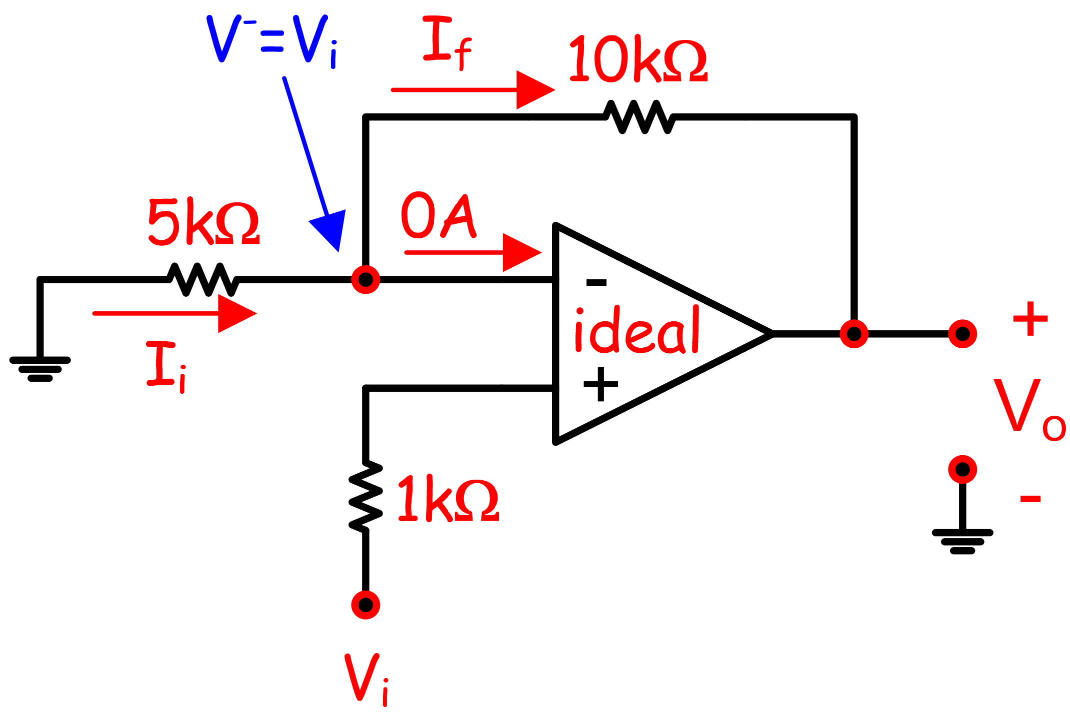
$$= \left(\frac{-10k\Omega}{2k\Omega}\right)(2v) = -5(2v) = \boxed{-10v}$$

$$I_i = I_f = \frac{V_i - V^-}{R_i} = \frac{2v - 0v}{2k\Omega} = \boxed{1mA}$$

$$R_{in} = R_i = \boxed{2k\Omega}$$

The "Non-inverting" op-amp

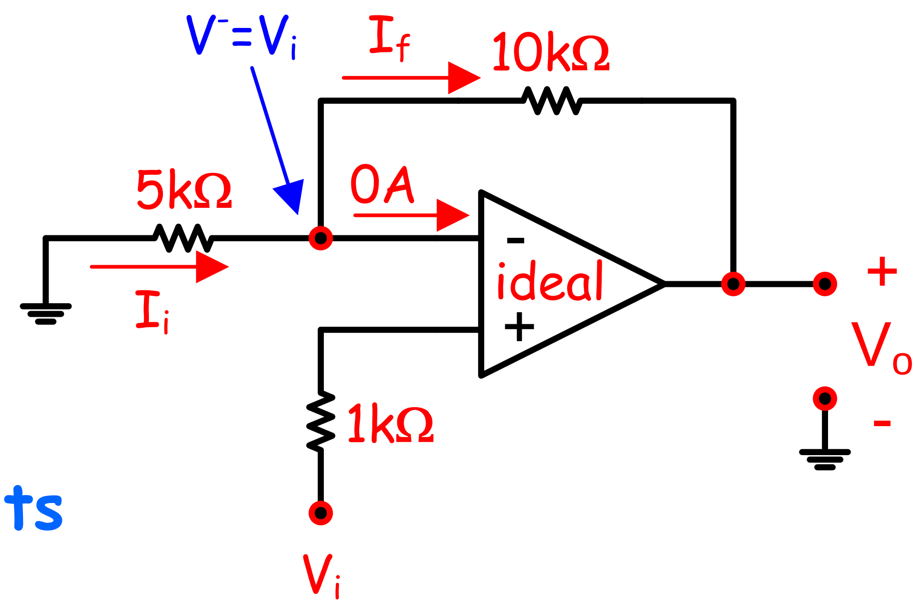
If you shift the input voltage down to the non-inverting terminal and ground R_i you now have a "Non-inverting" op-amp.



The gain of the Non-inverting op-amp is:

$$A_{cl} = 1 + \frac{R_f}{R_i}$$

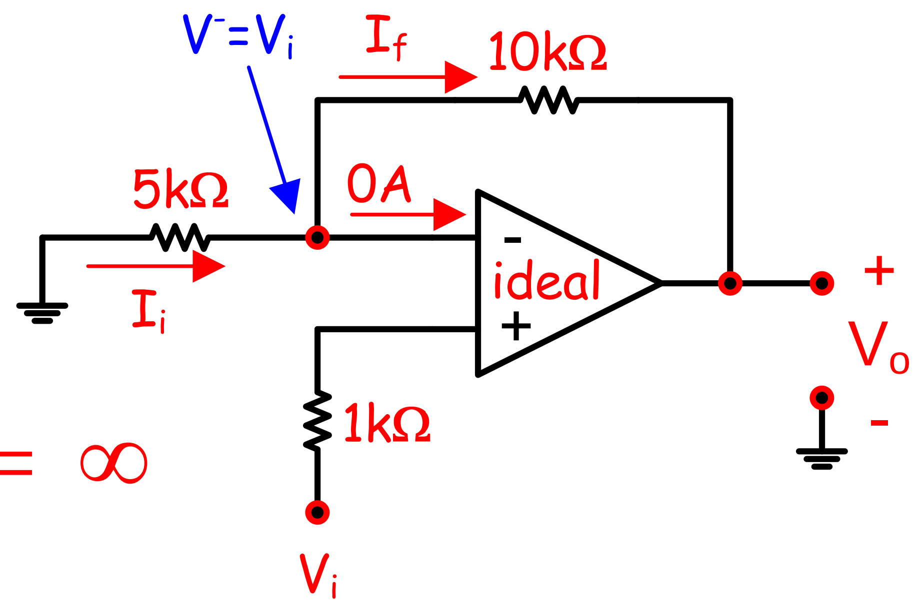
$$= \frac{R_f + R_i}{R_i}$$



Note that since there's 0A on the non-inverting terminal, there will be **0 volts across the resistor** so the voltage on the inverting terminal will be V_i.

The “Non-inverting” op-amp continued)

The input resistance of an ideal non-inverting op-amp will be infinite.



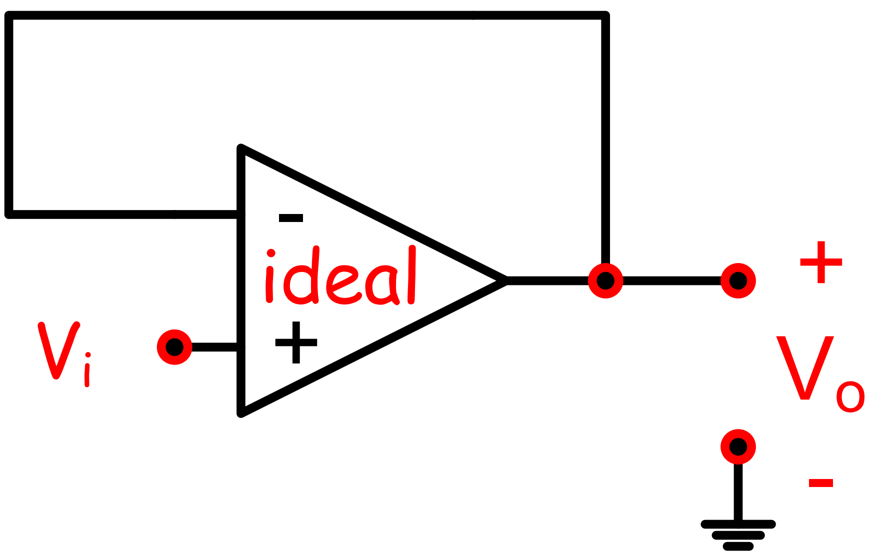
$$R_{in} = \infty$$

The "Voltage Follower"

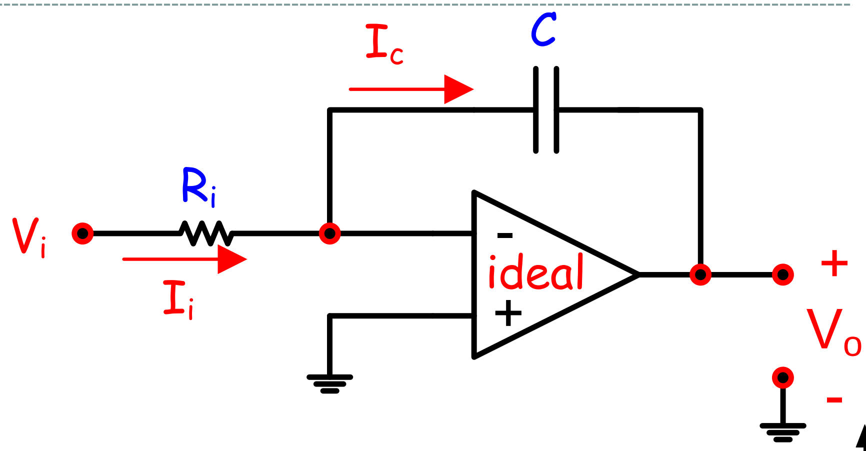
A Voltage follower is a special case of the "Non-inverting" op-amp with a voltage gain of 1.

$$A_{CL} = 1$$

$$R_{in} = \infty$$



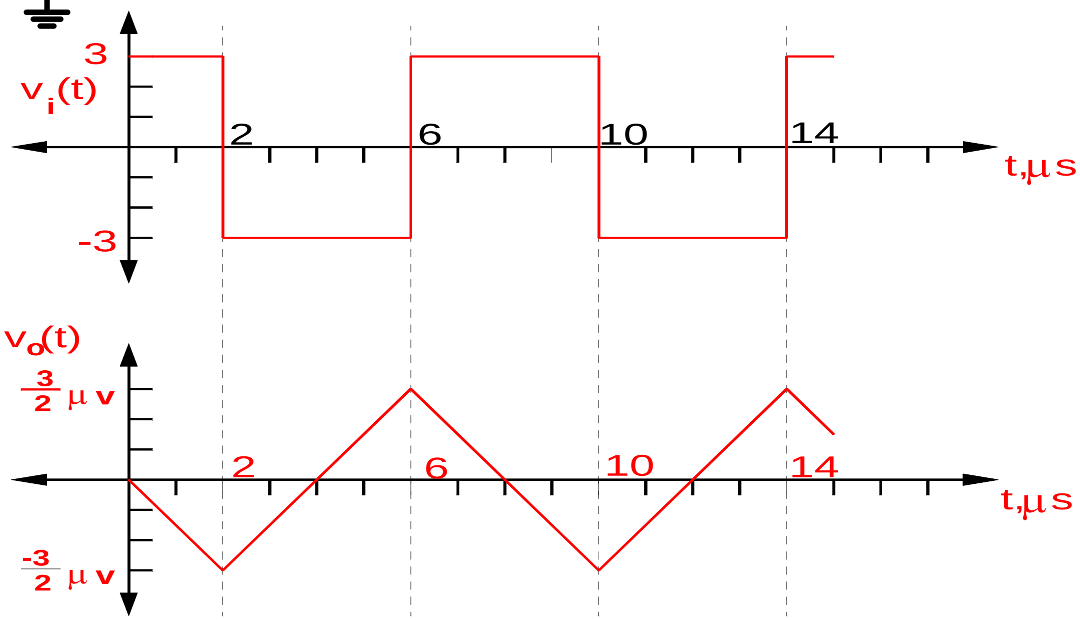
The “Ideal” Op-amp Integrator



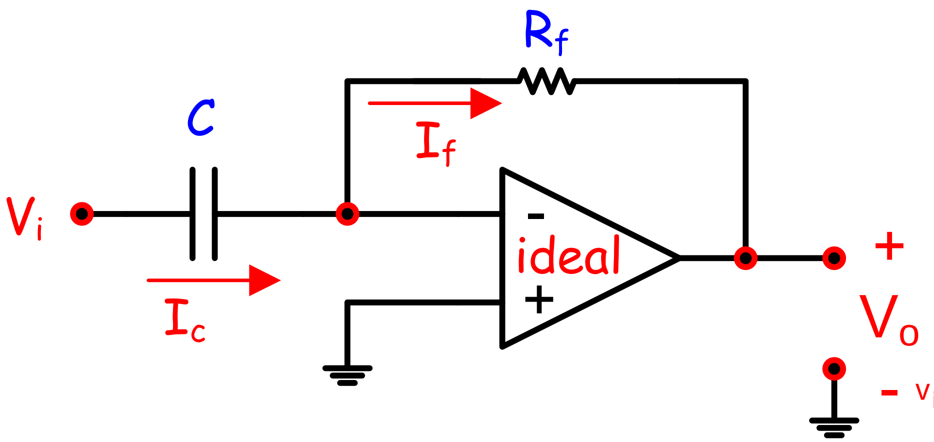
$$V_o(t) = \frac{-1}{R_i C}\int_0^t V_i dt + V_o(0)$$

with: R=10kΩ

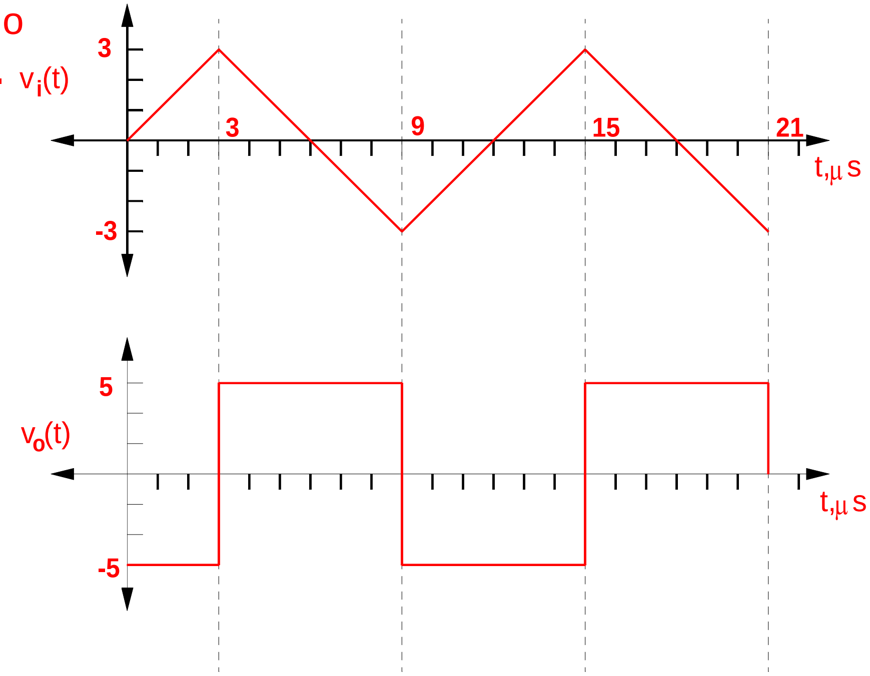
C=400 μ f



The “Ideal” Op-amp Differentiator



$$V_o(t) = -R_f C \frac{dV_i}{dt}$$



with

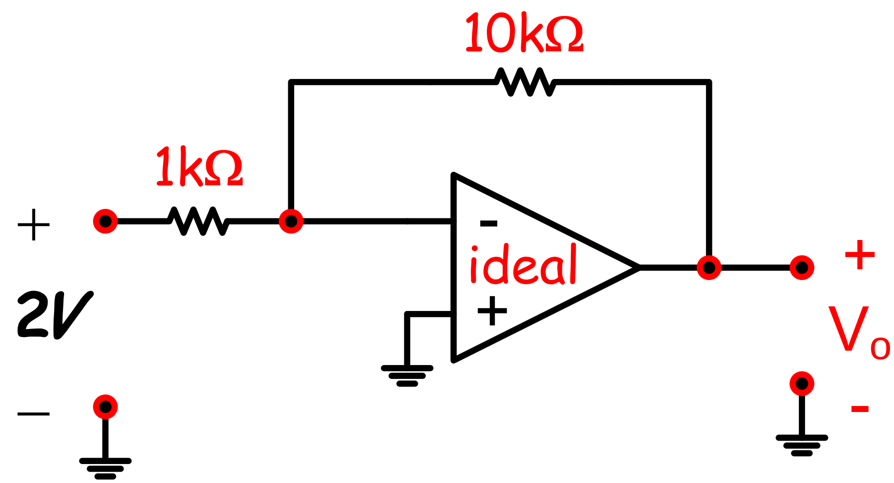
$$R = 50k\Omega$$

$$C = .1nf$$

Example

- What is the output voltage, V_o, of the ideal op-amp circuit shown?

The circuit is configured as An "Inverting Op-amp".



$$V_o = \frac{-R_f}{R_i} V_i = \frac{-10k\Omega}{1k\Omega}(2V) = \boxed{-20V}$$

Example

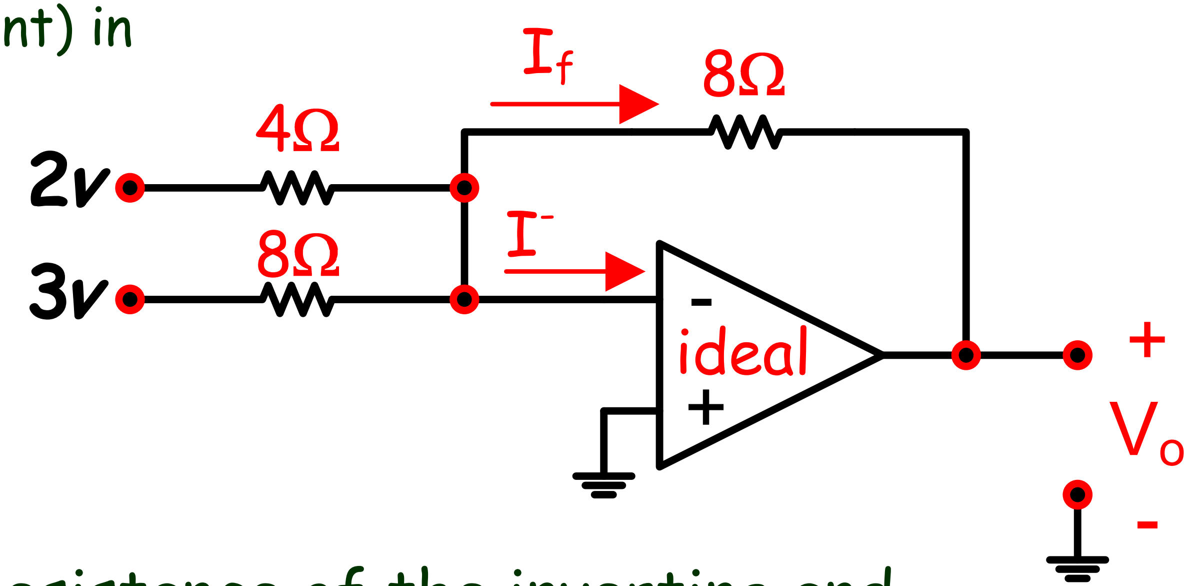
- What is the current, I^-, (inverting terminal current) in the circuit shown?

a) -0.88A

b) -0.25A

c) 0A

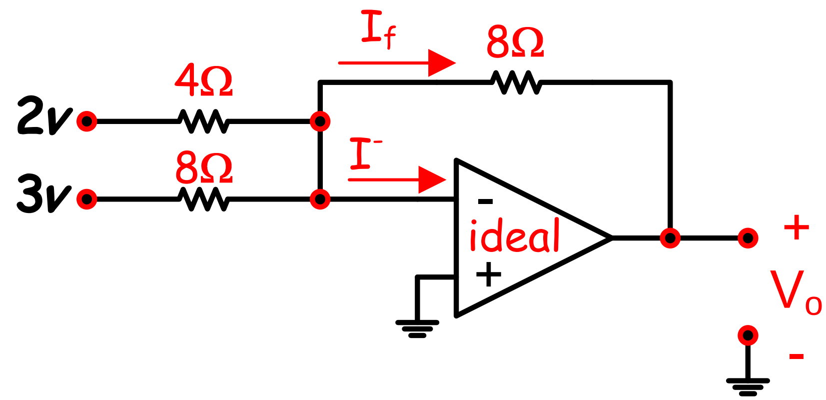
d) 0.25A



Since the input resistance of the inverting and non-inverting inputs is ideally infinite, the current into those inputs is ideally 0A. So the answer is: **c) 0A**

Example

- What is the output voltage of the circuit shown?



$$V_o = \left(\frac{-R_f}{R_1}\right)V_1 + \left(\frac{-R_f}{R_2}\right)V_2$$

$$= \left(\frac{-8\Omega}{4\Omega}\right)2v + \left(\frac{-8\Omega}{8\Omega}\right)3v$$

$$= (-2)\,2v + (-1)\,3$$

$$V_o = -4 - 3 = \boxed{-7v}$$

The circuit is an op-amp "**summing circuit**" or a "**Linear Combination Circuit**".

Example

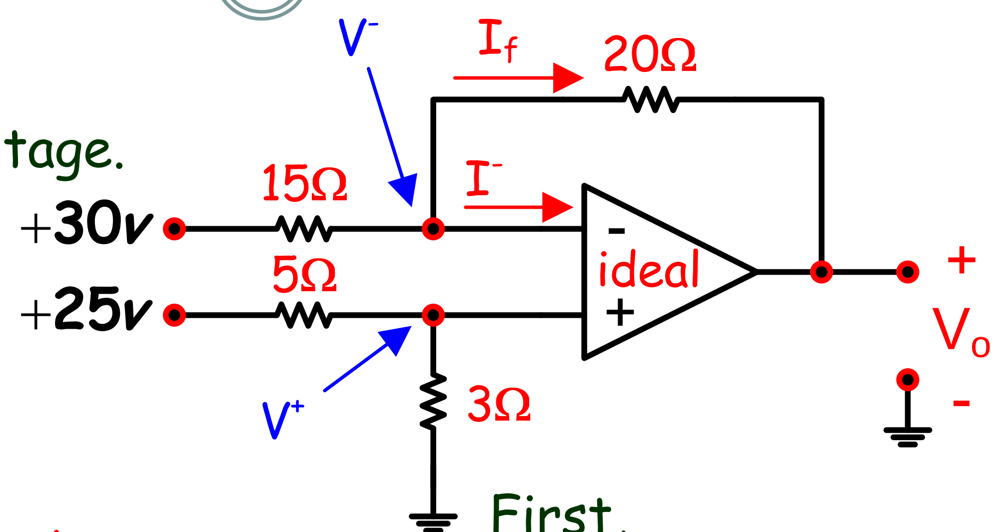
- For the "Difference Amplifier circuit shown, determine the output voltage.

a) -28.3v

b) -6.07V

c) 15.45v

d) -18.13v



We haven't talked about a "difference amp" but it can be easily solved with KCL, the Virtual short, and Ohm's law.

$$V^{+} = \left(\frac{3\Omega}{5\Omega + 3\Omega}\right) 25v = 5v$$

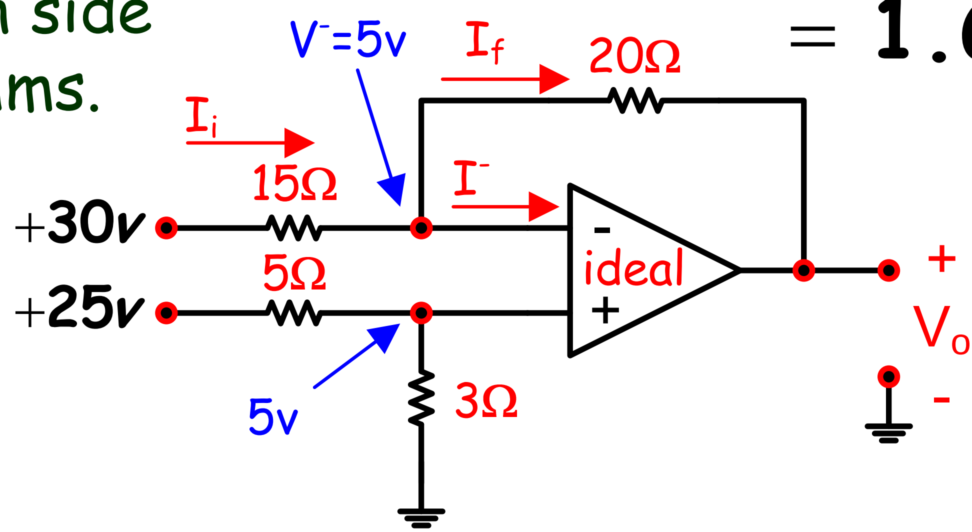
First,
the 5v in the equation is the voltage on the non-inverting leg "voltage divided" between the 5 and the 3 ohm resistors to give us V+.

Continued on the next page

(Example continued)

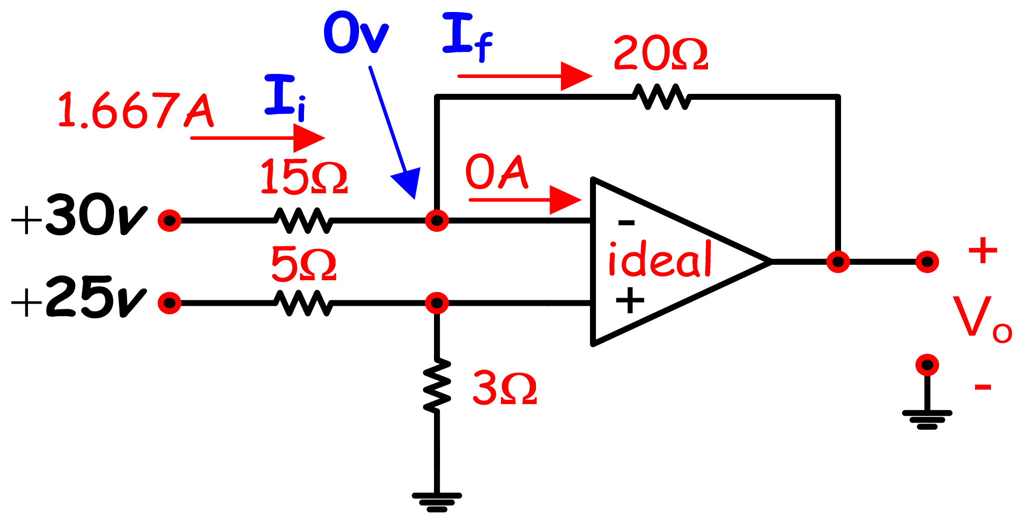
Next, due to the **virtual short**, the inverting terminal also has 5 volts on it. Now we can determine I_i, which is the difference between the voltages on each side divided by 15 ohms.

$$I_i = \frac{V_i - V^-}{15\Omega} = \frac{30v - 5v}{15\Omega} = 1.667A$$



(Example continued)

Now we note that since the current into the inverting terminal is zero, I_f = 1.667A by KCL.

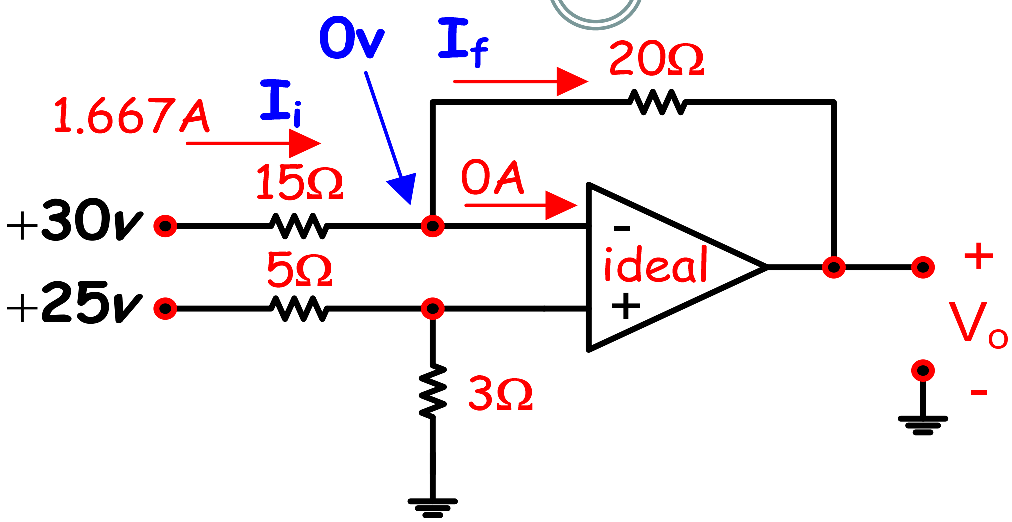


$$0 = -I_i + I^- + I_f$$

$$0 = -1.667A + 0A + I_f$$

$$I_f = 1.667A$$

(Example continued)



Finally we write an equation for Vo in terms of I_f.

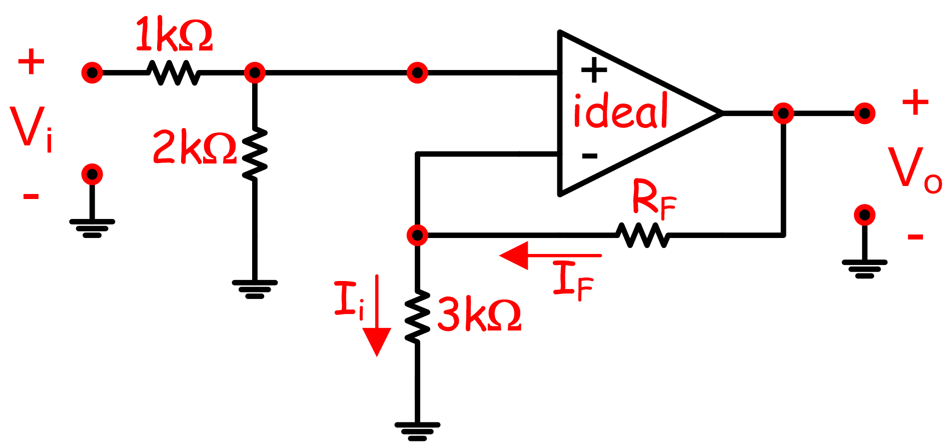
$$I_f = \frac{V^- - V_o}{20\Omega}$$

$$1.667A = \frac{5v - V_o}{20\Omega}$$

$$V_o = 5v - (1.667A)20\Omega = 5v - 33.333v = \boxed{-28.33v}$$

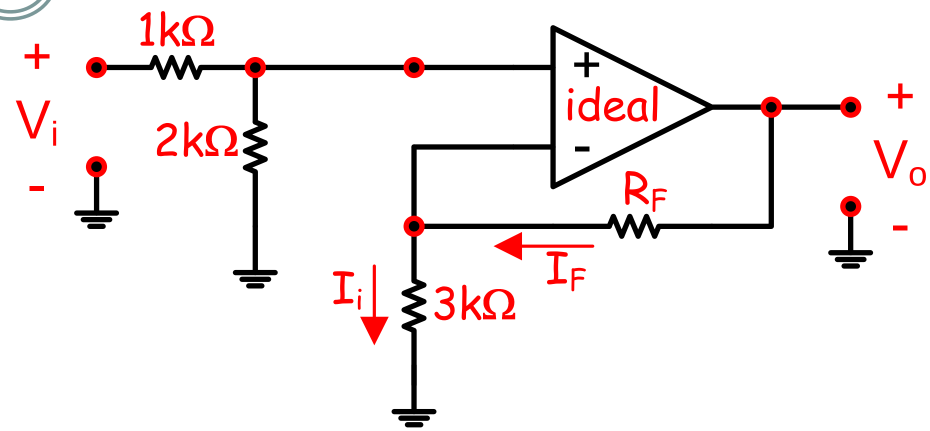
Example

For the ideal operational amplifier below, what should the value of R_f be in order to obtain a gain of 5?



Example continued on the next page.

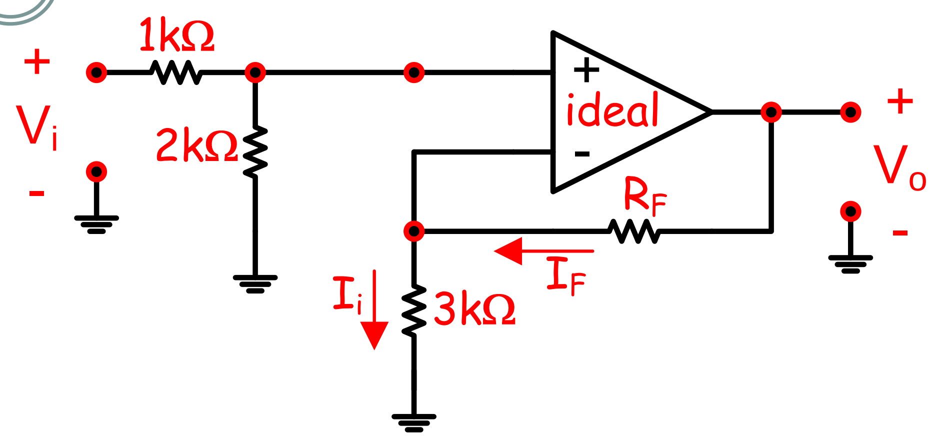
(Example continued)



First, note that the op-amp is upside down from the way we normally look at it. This is the way that the exam seems to show it a Lot, but it is not 'industry standard'. So, in the next slide we will flip it over.

Example continued on the next page

(Example continued)



Now that it is flipped over, note that the voltage on the non-inverting terminal is a voltage 'divided' V_i.

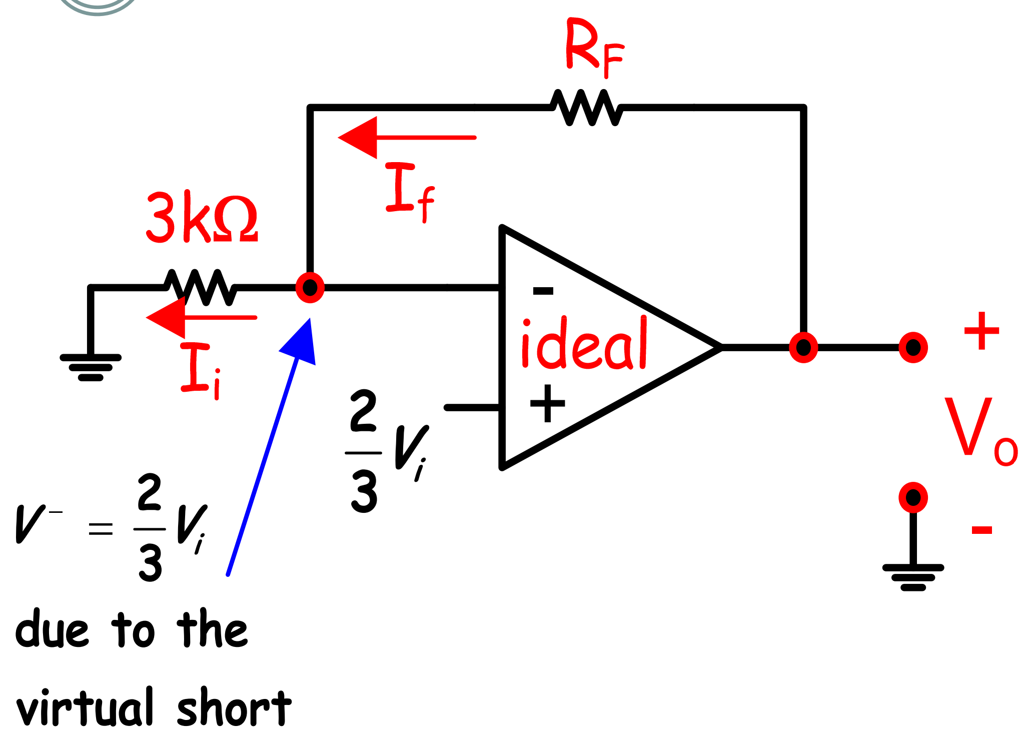
$$V^+ = \frac{2k\Omega}{1k\Omega + 2k\Omega} = \frac{2}{3}V_i$$

Example continued on the next page

(Example continued)

The voltage divider has been replaced by the result of the divider.

Next, note that due to the "**virtual short**" the voltage on the inverting terminal, V^- is also $2/3\ V_i$.

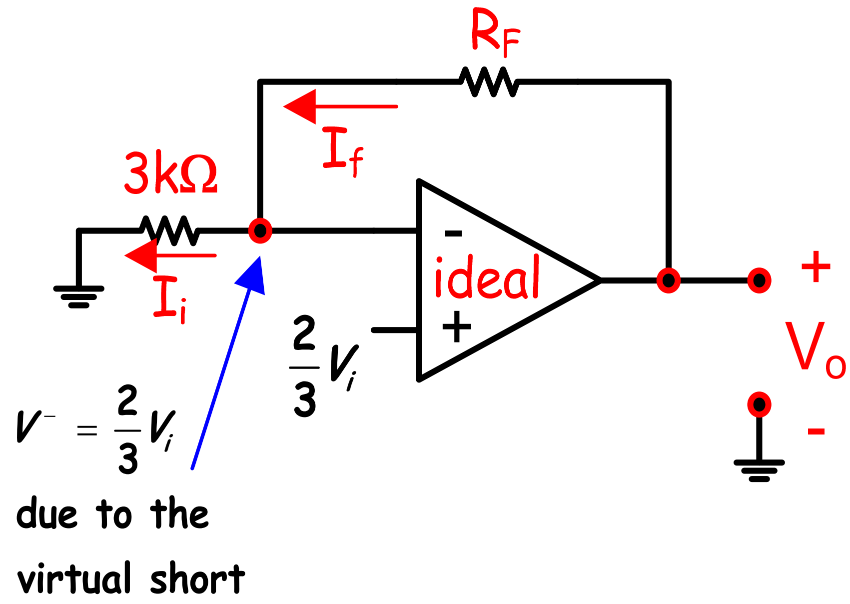


Example continued on the next page

(Example continued)

We can now find an equation for I_i. The voltage on the "**butt**" of the current arrow is $2/3\ V_i$ and the voltage at the tip of the current arrow is 0 volts.

Divide that "**difference voltage**" by 3k ohms and you get I_i.



$$I_i = \frac{V^- - 0}{3k\Omega} = \frac{V^-}{3k\Omega} = \frac{\frac{2}{3}V_i}{3k\Omega}$$

Example continued on the next page

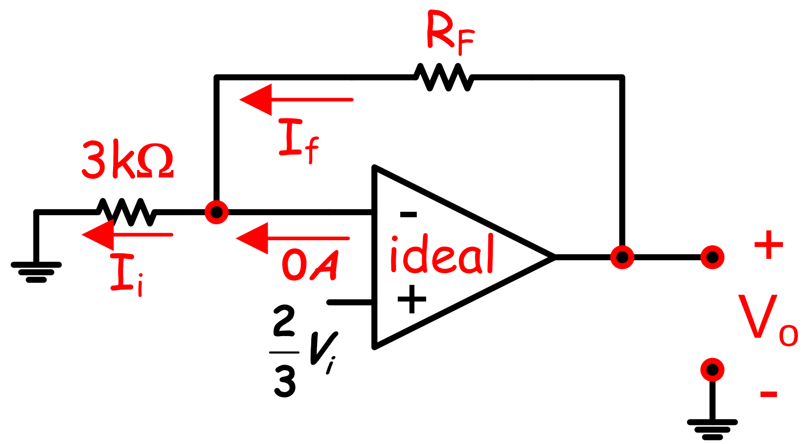
(Example continued)

By KCL,

$$0 = -I_f + I_i + I^-$$

$$0 = -I_f + I_i + 0$$

$$I_f = I_i$$

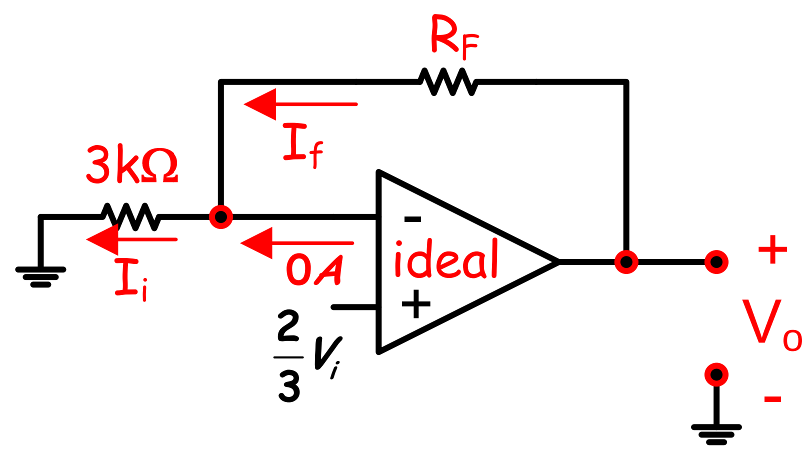


Now, remember that the INFINITE impedance of the Inverting and non-inverting terminals means that the current out of those terminals is "assumed" to be zero. Using this and KCL (above) we find that the feed back current, I_f is equal to I_i.

Problem continued on the next page.

(Example continued)

$$I_i = I_f = \frac{V_o - V^-}{R_f}$$



Now we can define this current in terms of V_o. The "butt" of the arrow is V_o while voltage at the tip of the arrow is $(2/3)V_i$.

Problem continued on the next page.

(Example continued)

The problem statement states that the gain is 5. So, that means that $V_o=+5V_i$

$$I_i = I_f = \frac{V_o - V^-}{R_f}$$

$$I_i = I_f = \frac{+5V_i - \frac{2}{3}V_i}{R_f}$$

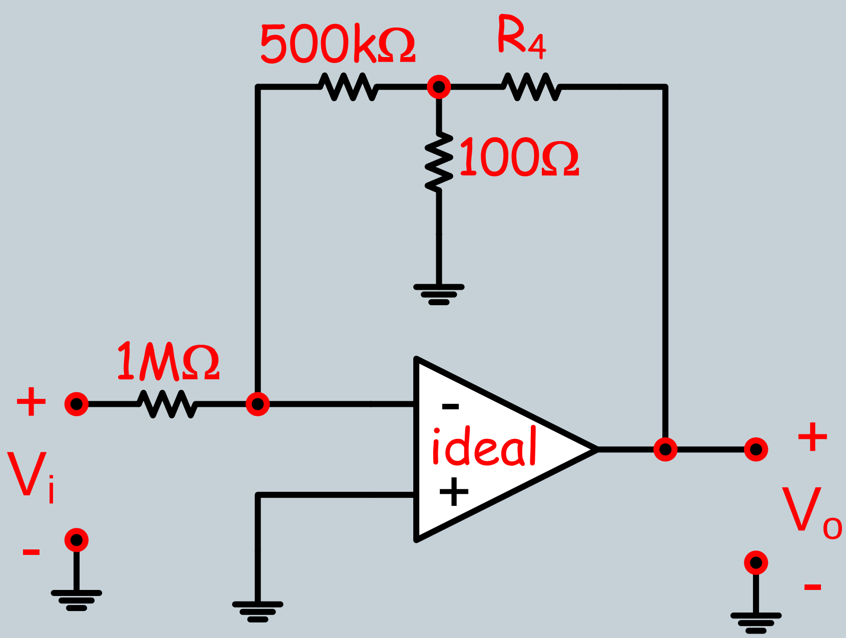
$$\frac{\frac{2}{3}V_i}{3k\Omega} = \frac{5V_i - \frac{2}{3}V_i}{R_f}$$

$$\frac{\frac{2}{3}}{3k\Omega} = \frac{5-\frac{2}{3}}{R_f} \Rightarrow R_f = \left(5-\frac{2}{3}\right)4.5k\Omega = \boxed{19.5k\Omega}$$

So, the answer to the question is that an R_f value of 19.5k ohms will result in a gain of 5.

Example

Evaluate the following amplifier to determine the value of R_4 required to obtain a voltage gain $\left(\frac{V_o}{V_i}\right)$ of -120.



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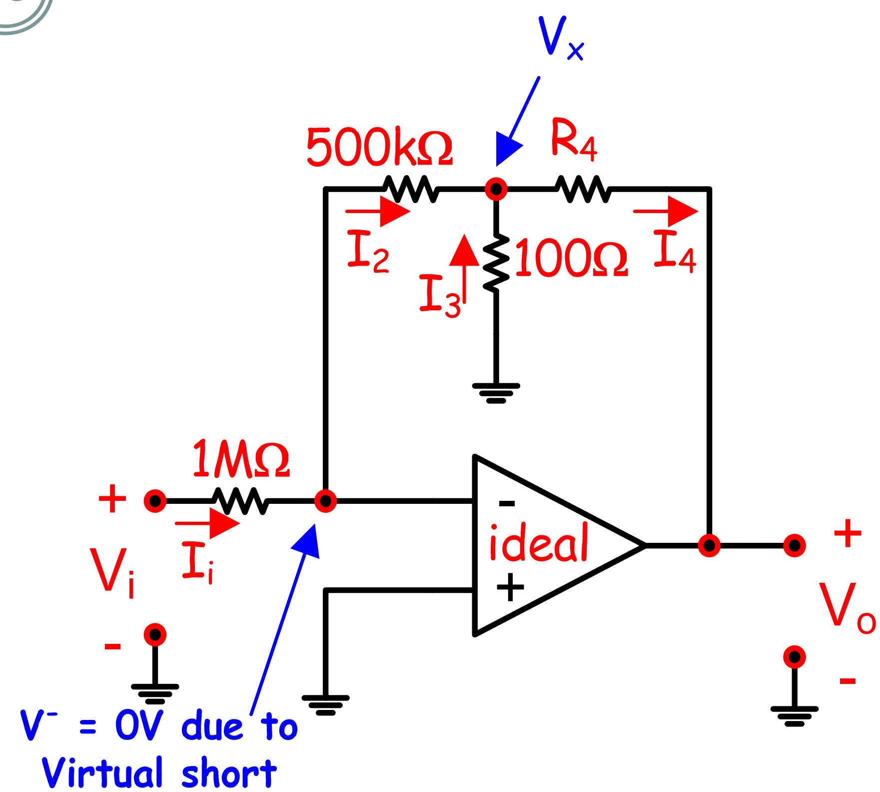
(Example continued)

Note that the inverting terminal voltage is 0 volts due to virtual short which exists between it and the non-inverting terminal.



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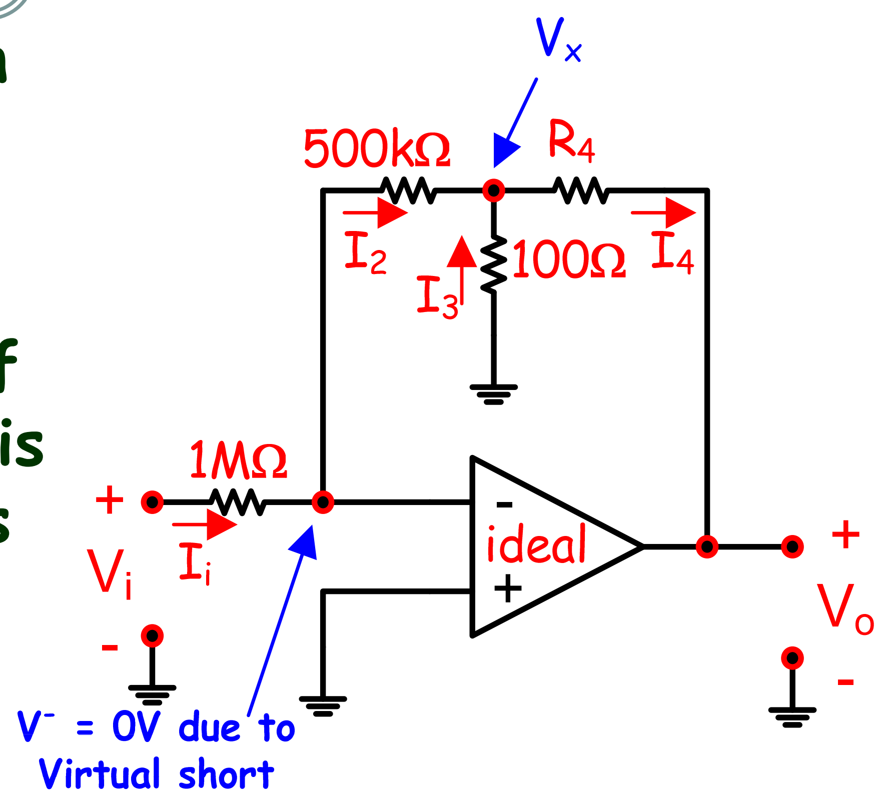
(Example continued)

We can now find an equation for the current I_i. Note that the voltage at the "**butt**" of the arrow is $+V_i$ and the voltage at the **tip** of the arrow is 0v. We use this voltage difference and Ohms law to find an equation for I_i.



$$I_i = \frac{V_i - 0v}{1M\Omega} = \frac{V_i}{1M\Omega}$$

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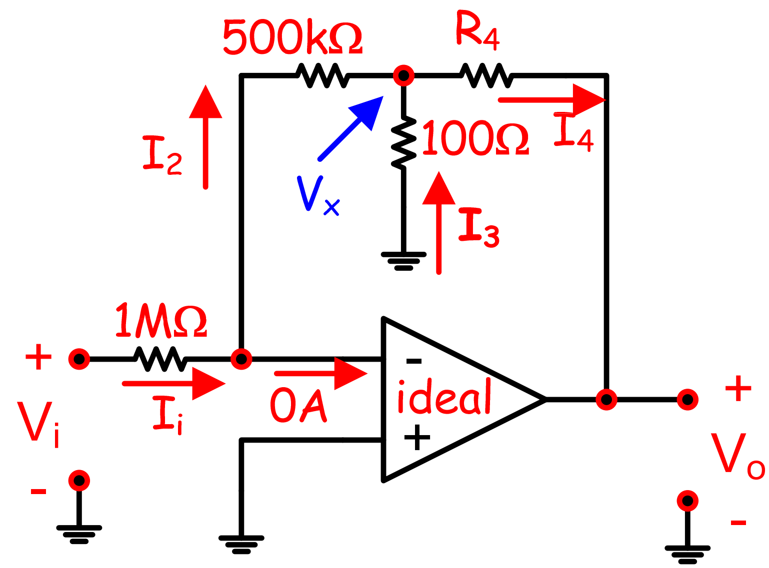
(Example continued)

Now, using the fact that the current into both the inverting and non-inverting terminals is "**assumed**" to be **0A** due to the "**assumed**" infinite input resistance, we can use **KCL** to show that I_2 equals I_i.



$$0 = -I_i + I_2 + I^-$$

$$0 = -I_i + I_2 + 0A$$

$$I_2 = I_i$$

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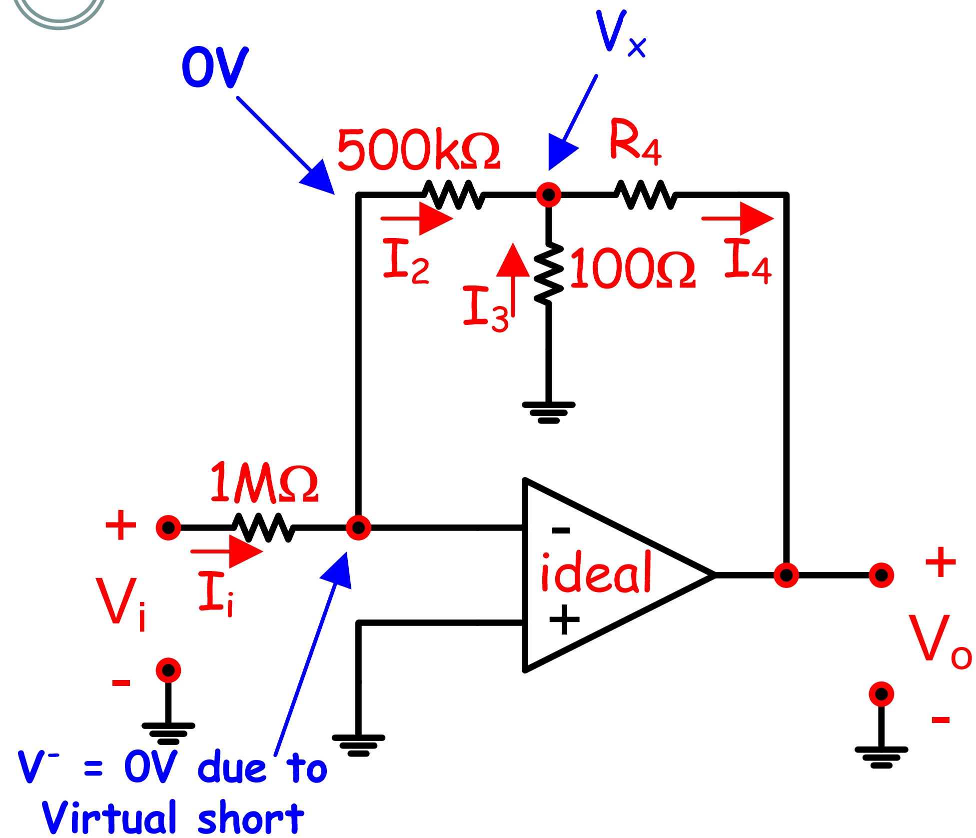
(Example continued)

We can use the same technique as before to define I_2 in terms of V_x, and then equate again to I_i.



$$I_2 = I_i = \frac{0v - V_x}{500k\Omega}$$

$$I_i = \frac{-V_x}{500k\Omega} \quad \text{so} \quad V_x = -I_i(500k\Omega)$$

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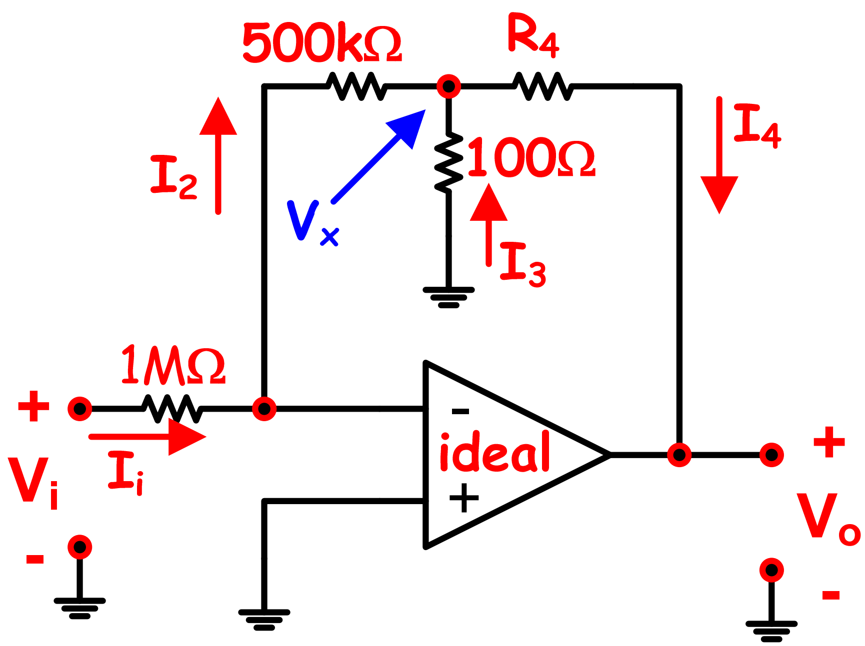
(Example continued)

Next define the current I_4 in terms of V_o and V_x.

$$I_4 = \frac{V_x - V_o}{R_4}$$



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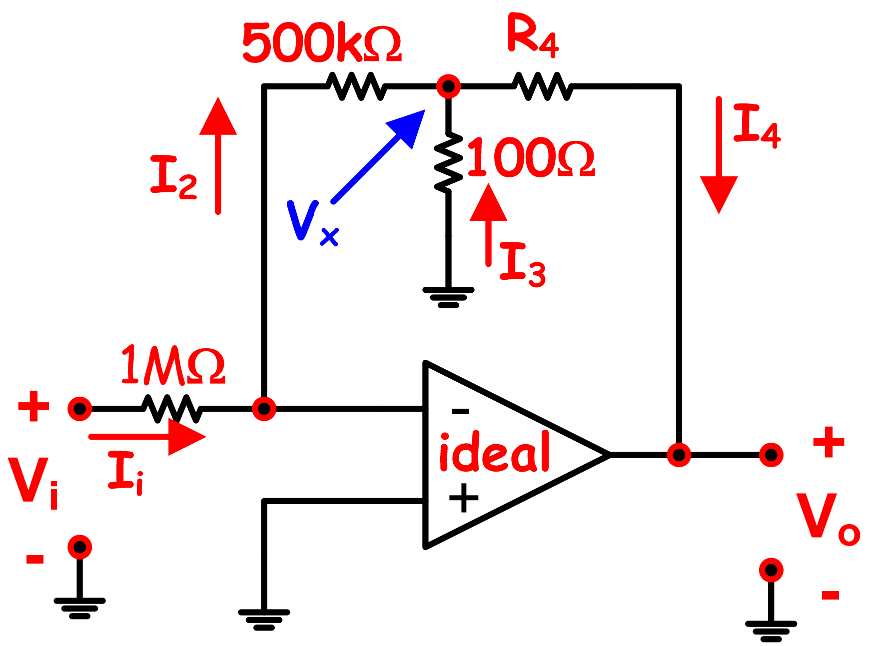
And now use KCL to define the currents in terms of voltage.



By KCL

$$0 = I_4 - I_3 - I_2$$

$$I_4 = I_3 + I_2$$

$$\frac{V_x - V_o}{R_4} = \frac{-V_x}{500k\Omega} + \frac{-V_x}{100\Omega}$$

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(Example continued)

From the problem statement:

$$\frac{V_o}{V_i} = -120 \therefore V_o = -120V_i$$

and $I_2 = I_1$ so we can expand that to

$$\frac{V_i}{1M\Omega} = \frac{-V_x}{500k\Omega} \Rightarrow\Rightarrow V_x = \left(\frac{-500k\Omega}{1M\Omega}\right)V_i$$

$$\frac{V_x - V_o}{R_4} = \frac{-V_x}{500k\Omega} + \frac{-V_x}{100\Omega}$$

$$\frac{\left(\frac{-500k\Omega}{1M\Omega}\right)V_i - \left(-120V_i\right)}{R_4} = \frac{-\left(\frac{-500k\Omega}{1M\Omega}\right)V_i}{500k\Omega} + \frac{-\left(\frac{-500k\Omega}{1M\Omega}\right)V_i}{100\Omega}$$

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(Example continued)

$$\frac{\left(\frac{-500k\Omega}{1M\Omega}\right)-(-120)}{R_4}=\frac{-\left(\frac{-500k\Omega}{1M\Omega}\right)}{500k\Omega}+\frac{-\left(\frac{-500k\Omega}{1M\Omega}\right)}{100\Omega}$$

$$1M\Omega\bullet\left[\frac{\left(\frac{500k\Omega}{1M\Omega}\right)+(-120)}{R_4}=\frac{\left(\frac{-500k}{1M}\right)}{500k\Omega}+\frac{\left(\frac{-500k}{1M}\right)}{100\Omega}\right]$$

$$\frac{500k\Omega-120M\Omega}{R_4}=\frac{-500k\Omega}{500k\Omega}+\frac{-500k\Omega}{100\Omega}$$

$$R_4=\frac{-119.5M\Omega}{-1-5k}=\frac{119.5M\Omega}{1+5k}=\boxed{23.895k\Omega}\approx 24k\Omega$$

So, the value of R_4 required to have a **voltage gain of -120** is **24k ohms**.