FE Review

AC CIRCUIT ANALYSIS

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AC ANALYSIS

2

Values:
- Peak,
- RMS,
- Effective
- Reactance
- Admittance
- Impedance
- Phasors

- Maximum Power Xfer
- Series
- Parallel
- Series and Parallel Resonance
- Transformers
- Impedance Matching Complex Power

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Peak Values

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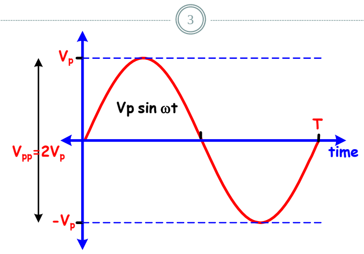

$$V(t) = 10\sin(1kt + 30°)$$

$$V_{peak} = |V| = 10V$$

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RMS Values

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$$i(t) = 5mA \sin\left(1kt + 30^\circ\right)$$

$$I_{peak} = 5mA$$

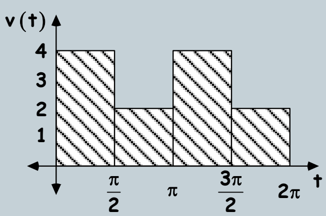
$$\bar{i}_{rms} = \frac{5mA\measuredangle 30^\circ}{\sqrt{2}}$$

$$= \boxed{3.536mA\measuredangle 30^\circ}$$

Example

5

Find the effective value of the voltage for the periodic waveform shown



Example continued on the next slide.

Example (continued)

6

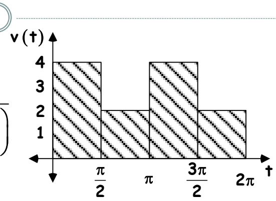
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\left(\int_0^{T/2}(4)^2\,dt + \int_{T/2}^{T}(2)^2\,dt\right)}$$

$$= \sqrt{\frac{1}{T}\left(16t\,\Big|_0^{T/2} + 4t\,\Big|_{T/2}^{T}\right)}$$

$$= \sqrt{\frac{1}{T}\left(16\left(T/2 - 0\right) + 4\left(T - T/2\right)\right)}$$

$$= \sqrt{\frac{1}{T}\left(8T + 2T\right)} = \sqrt{\frac{1}{T}\left(10T\right)} = \sqrt{10} = \boxed{3.16}$$



Capacitive and Inductive Reactance

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$$X_c = \frac{1}{2\pi fC}$$

$$C = 5\mu f \quad T = 1ms \therefore f = \frac{1}{T} = \frac{1}{1ms} = 1khz$$

$$X_c = \frac{1}{2\pi(1khz)(5\mu f)} = 31.83\Omega$$

$$X_L = 2\pi fL$$

$$L = 2mH$$

$$X_L = 2\pi(1khz)(2mH) = 12.57\Omega$$

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Conductance and Admittance

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$$\text{Conductance} = G = \frac{1}{R} = \frac{1}{10\Omega} = 100mS$$

$$\text{Admittance} = Y = \frac{1}{X}$$

$$Y_L = \frac{1}{X_L} = \frac{1}{j12.57\Omega} = -j79.55mS$$

$$Y_c = \frac{1}{X_c} = \frac{1}{-j31.83\Omega} = j31.42mS$$

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Impedance (Z)

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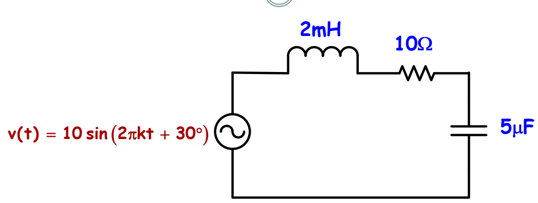

$$Z_T = R + jX_L - jX_C$$

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AC Circuit (Phasor Domain) Example

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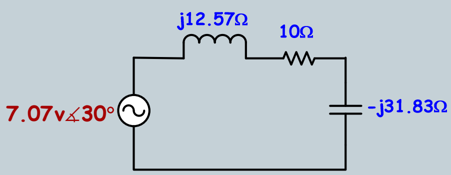
Analyze the following series AC circuit



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Example (continued)

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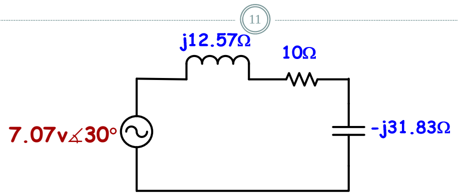

$$
\begin{aligned}
Z_T &= R + jX_L - jX_C \\
&= 10 + j12.57\Omega - j31.83\Omega \\
&= 10 - j19.26\Omega \\
&= 21.7\measuredangle -62.56^\circ
\end{aligned}
$$

Example continued on the next slide.

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Example (continued)

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$$I_T = \frac{V_T}{Z_T} = \frac{7.07V\measuredangle 30^\circ}{21.7\measuredangle -62.56^\circ} = 325mA\measuredangle 92.56^\circ$$

$$V_C = \frac{V_T X_C}{Z_T} = \frac{7.07V\measuredangle 30^\circ(31.83\measuredangle -90^\circ)}{21.7\measuredangle -62.56^\circ}$$

$$= \frac{225.0\measuredangle -60^\circ}{21.7\measuredangle -62.56^\circ} = 10.37\measuredangle 2.56^\circ$$

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Example

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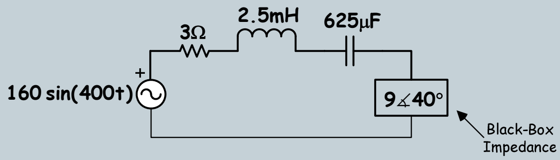
Analyze the following series AC circuit.
Note the use of a "Black Box".



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Example (continued)

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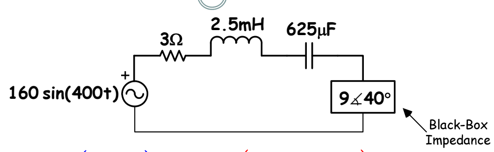

$X_L = \omega L = 400\,(2.5\text{mH}) = 1\Omega \qquad (Z_L = 1\Omega\angle 90^\circ)$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{400\,(625\mu\text{F})} = 4\Omega \qquad (Z_C = 4\Omega\angle -90^\circ)$

$Z_T = 3 + j1 - j4 + 9\angle 40^\circ$

$= 3 - j3 + 6.89 + j5.785$

$= 9.89 + j2.785 = \boxed{10.275\Omega\angle 15.727^\circ}$

Example continued on the next slide.

Example (continued)

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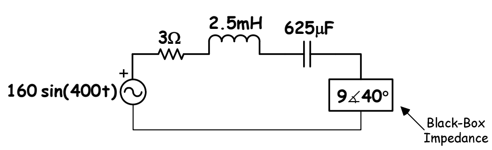

$\overline{V} = 160\angle 0^\circ\text{V}$

$\overline{I} = \dfrac{V}{Z_T} = \dfrac{160\angle 0^\circ\text{V}}{10.275\angle 15.727^\circ} = 15.725\text{A}\angle -15.725^\circ$

$i(t) = 15.725A\,\sin(400t - 15.725^\circ)$

Example continued on the next slide.

Example (continued)

16

$\bar{V} = 160V\angle 0°$

$\bar{I} = 15.725A\angle -15.725°$

$Z_T = 10.275\Omega\angle 15.727° = 9.89\Omega + j2.785\Omega$

The Circuit is INDUCTIVE in Nature

ELI the ICE person

ELI=> In an Inductive circuit, E leads I

ICE => In a Capacitive circuit, I leads E

The current lags the voltage in this circuit

Example continued on the next slide.

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Example (continued)

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The voltage across the inductor is:

$$V_L = IZ_L$$
$$= 15.725A\angle -15.725°(1\angle 90°)$$
$$= 15.725V\angle 74.275°$$

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Parallel Example

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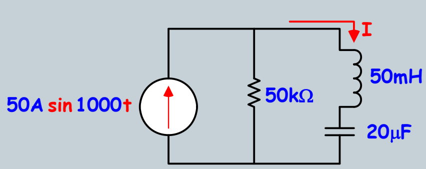
Find the Z_T of the circuit shown



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Parallel Example (continued)

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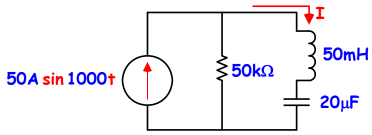

$$X_L = \omega L = 1000\left(50mH\right) = 50\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000\left(20\mu F\right)} = 50\Omega$$

$$Z_L + Z_C = j50 + -j50 = \boxed{0\Omega} \Leftarrow !!!$$

Example continued on the next slide.

Parallel Example (continued)

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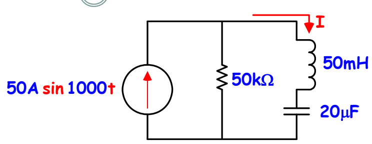

So, that leg is shorting out the other leg so the total impedance of the circuit is equal to 0.

A side note: Since the LC leg is shorting out the other leg then ALL the current is going thru it as well.

Example

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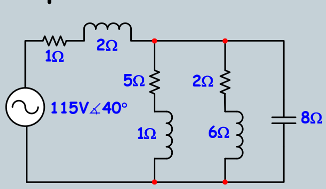
Find the total impedance and average power dissipated in the circuit shown



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Example (continued)

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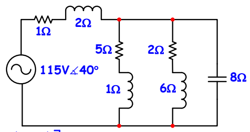

$$Z_T = (1 + j2) + [(5 + j1) \parallel (2 + j6) \parallel (-j8)]$$

$$= (1 + j2) + \left[\frac{1}{\frac{1}{(5 + j1)} + \frac{1}{(2 + j6)} + \frac{1}{-j8}}\right]$$

$$= 1 + j2 + 3.8621 + j1.0115$$

$$= 4.862 + j3.012 = \boxed{5.7192\angle 31.773^\circ}$$

Example continued on the next slide.

Example (continued)

23

$$Z_T = 4.862 + j3.012 = 5.72\angle 31.77^\circ$$

$$V = 115V\angle 40^\circ \text{ assumed to be RMS}$$

$$I = \frac{V}{Z_T} = \frac{115V\angle 40^\circ}{5.72\angle 31.77^\circ} = 20.1\angle 8.23^\circ \text{Arms}$$

$$P_{avg} = V_{rms} I_{rms} \cos\theta$$

$$= (115V)(20.1)\cos 31.77^\circ = 1.965\text{kW}$$

Power Triangle (Complex Power)

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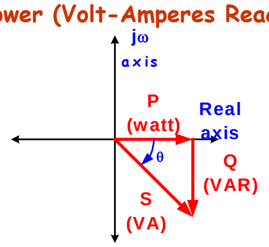
- True Power (Watts)
- Apparent Power (Volt-Amperes) or (VA)
- Reactive Power (Volt-Amperes Reactive) or (VARs)



Power Factor Angle

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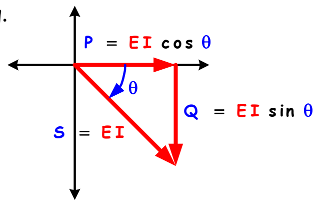
- The power factor angle (θ) goes FROM True Power TO the Apparent Power
- The magnitudes of the three powers can be found using Trig.



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Effective Values NOT Peak Values

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The title says it all. Convert from Peak to effective before you do anything!!!!!!

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Power Factor Calculation

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$$F_p = \cos(\theta) = \frac{P}{S}$$

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Example

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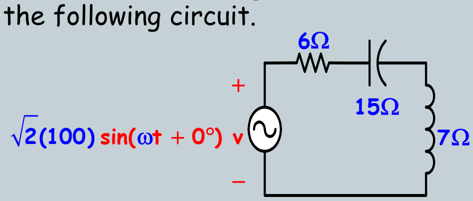
Find the Power Triangle for the following circuit.



Example continued on the next slide.

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Example (continued)

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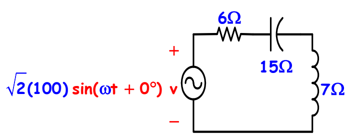
The example was kind enough to Represent peak voltage with a Square root of 2 showing!



$$E_{rms} = \frac{e(t)}{\sqrt{2}} = \frac{\sqrt{2}\,(100v)}{\sqrt{2}} = \boxed{100v}$$

Example continued on the next slide.

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Example (continued)

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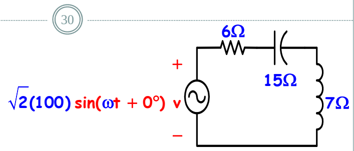

$$E_{rms} = 100v$$

$$Z_T = 6 - j15 + j7 = 6 - j8 = \boxed{10\Omega\angle -53.13^\circ}$$

$$I = \frac{E}{Z_T} = \frac{100v}{10\Omega\angle -53.13^\circ} = \boxed{10A\angle 53.13^\circ}$$

Example continued on the next slide.

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Example (continued)

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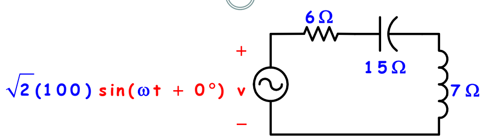

$V_R = IR = (10A\angle 53.13°)\,6\Omega = \boxed{60v\angle 53.13°}$

$V_C = IX_C = (10A\angle 53.13°)(15\Omega\angle -90°) = \boxed{150v\angle -36.87°}$

$V_L = IX_L = (10A\angle 53.13°)(7\Omega\angle 90°) = \boxed{70v\angle 143.13°}$

Example continued on the next slide.

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Example (continued)

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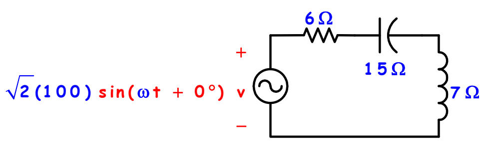

Note that since the capacitive reactance of the circuit is greater than the inductive reactance, the **Power factor** of this circuit will be **LEADING**.

Example continued on the next slide.

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Example (continued)

33

Note that since the capacitive reactance of the circuit is greater than the inductive reactance, the **Power factor** of this circuit will be **LEADING**.

ELI the ICE person:
Inductive circuits, voltage LEADS current.
Capacitive circuits, current LEADS voltage.
POWER FACTOR is a CURRENT relationship!

Example continued on the next slide.

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Example (continued)

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Power Factor $= F_p = \cos(\theta)$

$= \cos(53.13°) =$ **600m LEADING**

Total Apparent Power $= S_T = EI$

$= 100V(10A) =$ **1000 VA**

Total Reactive Power $= Q_T = S\sin(53.13°)$

$= (1000\ VA)(800m) =$ **800 VAR**

Example continued on the next slide.

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Example (continued)

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Power Factor = **600m LEADING**

Total Apparent Power = **1000 VA**

Total Reactive Power = **800 VAR**

Total True Power $= P_T = S\cos(53.13°)$

$= (1000\ VA)(600m) =$ **600 Watt**

Power Factor $= F_p = \frac{P}{S} = \frac{600}{1000} =$ **600m LEADING**

(Note that this is the same as earlier found)

Example continued on the next slide.

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Example (continued)

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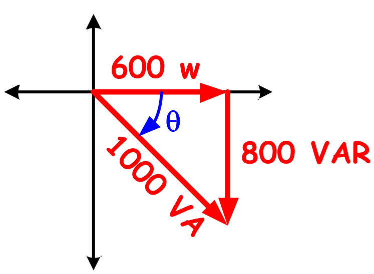

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Power Factor Correction

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The power company has very big interest in insuring that power supplied is available at peak efficiency. This implies that the current must be as low as possible. Thus, **Apparent Power (S) should be as low as possible** since the smaller the reactive power, the closer the apparent power will match true power.

$$S_T = EI_T = \sqrt{P_T^2 + Q_T^2}$$

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Example

38

A 5 hp motor with a 0.6 lagging power factor with an efficiency of 92% is connected to a 208-V, 60 hz supply. Find the Power Triangle.

Example continued on the next slide.

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Example (continued)

39

- **5 hp motor**
- **0.6 lagging power factor**
- **efficiency = 92%**
- **208-V, 60 hz supply.**

Most MOTORS (but not all) are inductive in nature. Since the power factor is lagging, this motor must be inductive!

Example continued on the next slide.

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Example (continued)

40

- 5 hp motor
- 0.6 lagging power factor
- efficiency = 92%
- 208-V, 60 hz supply.

$$1hp = 746W$$

$$P_o = 5hp = 5(746W) = 3730\ W$$

$$P_{in} = \frac{Po}{\eta} = \frac{3730}{.92} = 4054.348\ W$$

$$F_p = \cos(\theta) = .6 \quad \text{so} \quad \theta = \cos^{-1}(.6) = 53.13°$$

Example continued on the next slide.

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Example (continued)

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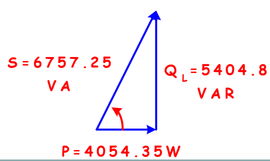
Since it was lagging, it was inductive,

Since $P = S\cos(\theta)$ then

$$S = \frac{P}{\cos(\theta)} = \frac{4054.348W}{.6} = 6757.246\ VA$$

$$Q = S\sin(\theta) = 6757.246VA\ \sin(53.13) = 5405.79\ VAR$$



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Example (continued)

42

Now that a Power Triangle has been found, we need to insert the required component in the circuit which will counteract the amount of reactive power to reduce the difference between the apparent and true power. Since the circuit is inductive in nature we need to add in a capacitance to the problem.

Example continued on the next slide.

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Example (continued)

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A net unity power factor level is needed by introducing a **capacitive reactive power level of 5405.8 VARs to balance Q_L**. Find the required C.

$$Q_C = \frac{V^2}{X_C} \quad \text{so}$$

$$X_C = \frac{V^2}{Q_C} = \frac{(208V)^2}{5405.8VAR} = 8\Omega$$

$$C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi(60)(8\Omega)} = \boxed{331.6\mu F}$$

Example continued on the next slide.

Example (continued)

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At $0.6F_p$ (before correction)

$$S = VI = 6757.25\ VA$$

and $$I = \frac{S}{V} = \frac{6757.25\ VA}{208V} = 32.49\ A$$

Example continued on the next slide.

Example (continued)

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At Unity F_p (after correction)

$$S = VI = 4054.35\ VA$$

and $$I = \frac{S}{V} = \frac{4054.35\ VA}{208V} = 19.49\ A$$

$$\frac{32.49A - 19.49\ A}{32.49A} * 100\%$$

= 40% reduction in current

Example continued on the next slide.

Example (continued)

46

In the motor (**before correction**), the current lagged the voltage by 53.13 degrees and the true power was 4054.35W

$$I_m = \frac{P}{E\cos(\theta)} = \frac{4054.35}{(208V)(.6)} = 32.49A$$

which gives us $I_m = 32.49A\measuredangle -53.13°$

Therefore,

$$Z_m = \frac{E}{I_m} = \frac{208V\measuredangle 0°}{32.49A\measuredangle -53.13°} = 6.4\measuredangle 53.13°$$

$$= \boxed{3.84\Omega + j5.122\Omega}$$

In parallel, it would be

$$Y = \frac{1}{6.4\measuredangle 53.13°} = .094S - j.125S = \boxed{\frac{1}{10.64\Omega} + \frac{1}{j8\Omega}}$$

Example continued on the next slide.

Example (continued)

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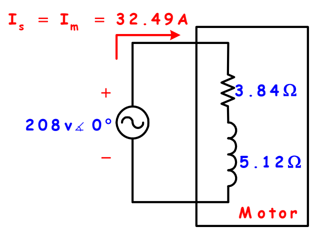
Motor (**before correction**)



Example continued on the next slide.

Example (continued)

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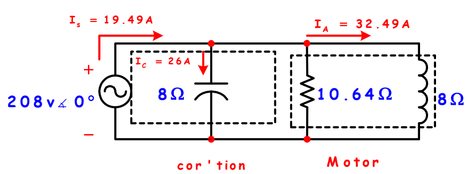
Earlier we calculated the parallel values of the motor and even earlier we found that the required additional C reactance 8 ohms.



Example continued on the next slide.

Example (continued)

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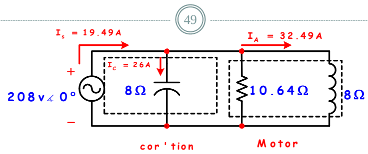

Since

$$Y_T = \frac{1}{-jX_C} + \frac{1}{R} + \frac{1}{+jX_L} = \frac{1}{R}$$

$$|I_s| = EY_T = E\left(\frac{1}{R}\right) = 208v\left(\frac{1}{10.64}\right) = 19.54A$$

$$|I_C| = \frac{E}{X_C} = \frac{208v}{8\Omega} = 26A$$

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Resonant Circuits

50

A resonant circuit has a zero degree current phase angle difference. This is the same thing as saying that the circuit is purely resistive (the power factor is equal to '1')

In order for this to occur, the inductive and the Capacitive reactance's must cancel each other out.

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Resonant Circuits

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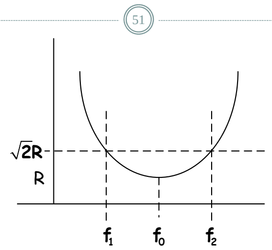

As a circuit approaches resonance, there are two points called the half-power points (where have the power is dissipated in the resistor. The freq difference between the half power points is the BW.

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Resonant Circuits

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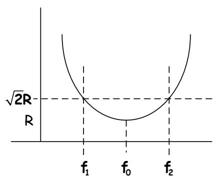

$BW = f_2 - f_1$

Resonant Freq $= f_0$

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Quality Factor (Q)

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The quality factor, Q, for a resonant circuit is a dimensionless value which compares reactive energy stored in the reactive elements to the resistive energy dissipated.

$$Q = \frac{f_0}{BW} = \frac{X_T}{R}$$

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Series Resonant Circuits

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Impedance is at a minimum
Impedance equals R
Current and Voltage are in phase
Current is maximum
Power dissipation is maximum

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Series Resonant Circuit Example

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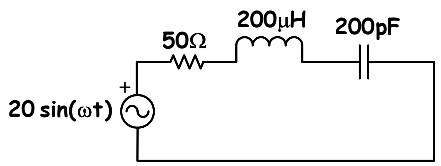

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{200\mu(200p)}} = 5 \bullet 10^6 \ {}^{rad}\!/_{s}$$

Example continued on the next slide.

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Example (continued)

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half – power points are:

$$\omega_o \pm \frac{R}{2L} = 5M \pm \frac{50\Omega}{2(200\mu H)}$$

$$= 5.125M \ {}^{rad}\!/_{s} \quad \text{and} \quad 4.875M \ {}^{rad}\!/_{s}$$

The total circuit Z is just the resistance at resonance.

The resonant current is:

$$I_o = \frac{V}{Z_o} = \frac{V}{R} = \frac{20\measuredangle 0^\circ}{50\Omega} = 400mA\measuredangle 0^\circ$$

Example continued on the next slide.

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Example (continued)

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The voltages across the elements are:

$$V_R = I_o R = 400mA(50\Omega) = 20V\measuredangle 0^\circ$$

$$V_L = I_o X_L = 400mA(\omega_o)200\mu H$$

$$= j400mA\left(5M \ {}^{rad}\!/_{s}\right)200\mu H$$

$$= 400\Omega\measuredangle 90^\circ$$

$$V_C = I_o X_C = 400mA\left(\frac{-j}{400mA\left(5M \ {}^{rad}\!/_{s}\right)200pF}\right)$$

$$= 400\Omega\measuredangle -90^\circ$$

Example continued on the next slide.

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Example (continued)

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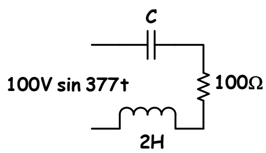
What C value is needed for the circuit to have a Power Factor = 1?

Power factor = 1 at Circuit resonance so,



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{(\omega_0)^2 L} = \frac{1}{(377)^2\, 2H} = 3.5\mu F$$

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Parallel Resonant Circuits

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Same as Series Resonant Circuits except that:
Current is at a minimum and
Power Dissipated is at a minimum.

At resonance:

$$X_L = X_C$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 RC = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$$

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Parallel Resonance Example

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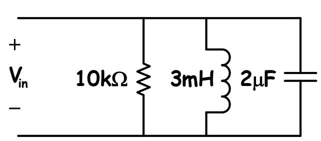
Calculate the resonant freq and the Q for the circuit below



Example continued on the next slide.

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Example (continued)

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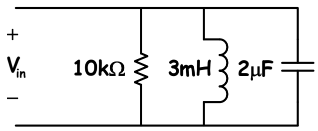
Calculate the resonant freq and the Q for the circuit below.



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(3mH)(2\mu f)}} = \frac{1}{\sqrt{6 \bullet 10^{-9}}} = 12.9k \, {}^{rad}\!/_{s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{10 \bullet 10^3}{12.9 \bullet 10^3 \, {}^{rad}\!/_{s} \, (3 \bullet 10^{-3} H)} = 258.4$$

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AC Maximum Power Transfer

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In DC circuits, in order for the Maximum power to be transfered to the load, $R_{load} = R_{thevenin}$.

In AC circuits, the requirement is slightly different.
In order for the maximum power to be transfered to the load:

$R_{load} = R_{Thev}$

$X_{load} = - X_{Thev}$

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