# FE Math Review and EET 300 Laplace Review

1

## LAPLACE TRANSFORMS



## Laplace Example 1

2

$$\mathcal{L}(s) = \frac{20}{s(s+10)}$$

What is the partial fraction expansion of $\mathcal{L}(s)$?

$$\frac{20}{s(s+10)} = \frac{A}{s} + \frac{C}{(s+10)}$$

$$A = \frac{20}{(s+10)}\Big|_{s=0} = \frac{20}{(0+10)} = 2$$

$$C = \frac{20}{s}\Big|_{s=-10} = \frac{20}{-10} = -2$$

$$\frac{20}{s(s+10)} = \boxed{\frac{2}{s} - \frac{2}{(s+10)}}$$



## Laplace Example 2

3

$$\mathcal{L}(s) = \frac{20}{s(s+10)} = \frac{2}{s} - \frac{2}{(s+10)}$$

What is the inverse Laplace transform of $\mathcal{L}(s)$?

$$\mathcal{L}^{-1}\left\{\frac{2}{s}\right\} = 2$$

$$\mathcal{L}^{-1}\left\{\frac{-2}{(s+10)}\right\} = -2e^{-10t}$$

$$\mathcal{L}^{-1}\left\{\frac{20}{s(s+10)}\right\} = \boxed{2 - 2e^{-10t}}$$

## Laplace Example 3

4

$$\mathcal{L}(s) = \frac{s(s+10)}{(s+5)(s+15)}$$

Before going any further, you need to ensure that the order of the denominator is greater than the numerator.

$$\frac{s^2+10s}{(s^2+20s+75)}$$ the order of the denominator is = to the order of the numerator

So, divide the numerator by the denominator.

$$\begin{array}{r|l} & 1 \\ s^2+20s+75 & s^2+10s+0 \\ & -(s^2+20s+75) \\ \hline & -10s-75 \end{array} \Rightarrow \boxed{1+\frac{-10s-75}{(s+5)(s+15)}}$$

8/24/2010

## Example 3 (cont)

5

$$\mathcal{L}(s) = 1 + \frac{-10s-75}{(s+5)(s+15)}$$

What is the partial fraction expansion of $\mathcal{L}(s)$?

$$1+\frac{-10s-75}{(s+5)(s+15)} = 1 + \frac{A}{(s+5)} + \frac{C}{(s+15)}$$

$$A = \frac{-10s-75}{(s+15)}\Big|_{s=-5}$$

$$A = \frac{-10(-5)-75}{(-5+15)}$$

$$A = \frac{50-75}{10} = \frac{-25}{10} = \boxed{-2.5}$$

8/24/2010

## Example 3 (cont)

6

$$1+\frac{-10s-75}{(s+5)(s+15)} = 1 + \frac{-2.5}{(s+5)} + \frac{C}{(s+15)}$$

$$C = \frac{-10s-75}{(s+5)}\Big|_{s=-15}$$

$$C = \frac{-10(-15)-75}{(-15+5)}$$

$$C = \frac{150-75}{-10} = \frac{75}{-10} = \boxed{-7.5}$$

$$\boxed{\mathcal{L}(s) = 1 + \frac{-2.5}{(s+5)} + \frac{-7.5}{(s+15)}}$$

8/24/2010

## Example 4

7

$$\mathcal{L}(s) = 1 + \frac{-2.5}{(s+5)} + \frac{-7.5}{(s+15)}$$

What is the inverse Laplace transform of $\mathcal{L}(s)$?

$$\boxed{1 - 2.5e^{-5t} - 7.5e^{-15t}}$$

8/24/2010

## Example 5

8

What is the inverse Laplace transform of

$$\mathcal{L}(s) = \frac{20s}{s^2 + 20s + 75}$$

The partial fraction expansion will be of the form

$$\frac{20s}{(s+5)(s+15)} = \frac{A}{(s+5)} + \frac{C}{(s+15)}$$

8/24/2010

## Example 5 (continued)

9

$$\frac{20s}{(s+5)(s+15)} = \frac{A}{(s+5)} + \frac{C}{(s+15)}$$

$$A = \frac{20s}{(s+15)}\Big|_{s=-5} = \frac{20(-5)}{(-5+15)} = \frac{-100}{10} = \boxed{-10}$$

$$C = \frac{20s}{(s+5)}\Big|_{s=-15} = \frac{20(-15)}{(-15+5)} = \frac{-300}{-10} = \boxed{30}$$

$$\mathcal{L}^{-1}\left\{\frac{-10}{(s+5)} + \frac{30}{(s+15)}\right\} = \boxed{-10e^{-5t} + 30e^{-15t}}$$

8/24/2010

## Example 6

10

What is the Laplace transform of the step function f(t) below?

$$f(t) = u(t-1) + u(t-2)$$

$$\boxed{F(s) = \frac{e^{-1s}}{s} + \frac{e^{-2s}}{s}}$$

8/24/2010

## Example 7

11

What is the Laplace transform of $(\sin t)(\cos t)$?

First, the function has to be seperated into something which is represented on the transform table.

Using the double angle formula for $\sin 2t$ we get:

$$\sin 2t = 2\sin t \cos t \quad \therefore \quad (\sin t)(\cos t) = \frac{\sin 2t}{2}$$

$$\mathcal{L}((\sin t)(\cos t)) = \frac{1}{2}\mathcal{L}(\sin 2t) = \frac{1}{2}e^{-\alpha s}\left(\frac{\omega}{s^2+\omega^2}\right) \text{ where } \begin{pmatrix}\alpha = 0 \\ \omega = 2\end{pmatrix}$$

$$= \frac{1}{2}e^{-0s}\left(\frac{2}{s^2+2^2}\right) = \boxed{\frac{1}{2}\left(\frac{2}{s^2+4}\right)} = \boxed{\frac{1}{s^2+4}}$$

8/24/2010

## Laplace Transform of Derivatives

12

$$L(y') = sy - y(0)$$

$$L(y'') = s^2y - sy(0) - y'(0)$$

8/24/2010

## Example 8

13

Solve the following DE with Laplace Transforms.

$0 = y'' + 4y' + 3y$

Boundary conditions $\begin{array}{l} y(0) = 3 \\ y'(0) = 1 \end{array}$

Use $\begin{array}{l} \mathcal{L}(y') = sy - y(0) \\ \mathcal{L}(y'') = s^2y - sy(0) - y'(0) \end{array}$

8/24/2010

## Example 8 (continued)

14

$$\begin{aligned} 0 &= y'' + 4y' + 3y \\ &= s^2y - sy(0) - y'(0) + 4[sy - y(0)] + 3y \\ &= s^2y - sy(0) - y'(0) + 4sy - 4y(0) + 3y \\ &= s^2y - s(3) - 1 + 4sy - 4(3) + 3y \end{aligned}$$

$$s^2y + 4sy + 3y = s(3) + 1 + 12$$

$$s^2y + 4sy + 3y = 3s + 13$$

$$(s+1)(s+3)y = 3s + 13$$

$$y = \frac{3s+13}{(s+1)(s+3)}$$

8/24/2010

## Example 8 (cont)

15

$$y = \frac{3s+13}{(s+1)(s+3)} = \frac{A}{(s+1)} + \frac{B}{(s+3)}$$

$$A = \frac{3s+13}{(s+3)}\Big|_{s=-1} = \frac{3(-1)+13}{((-1)+3)} = \frac{10}{2} = \boxed{5}$$

$$B = \frac{3s+13}{(s+1)}\Big|_{s=-3} = \frac{3(-3)+13}{((-3)+1)} = \frac{4}{-2} = \boxed{-2}$$

$$\mathcal{L}(s) = \frac{5}{(s+1)} + \frac{-2}{(s+3)}$$

$$\mathcal{L}^{-1}(s) = \boxed{5e^{-1t} - 2e^{-3t}}$$

8/24/2010