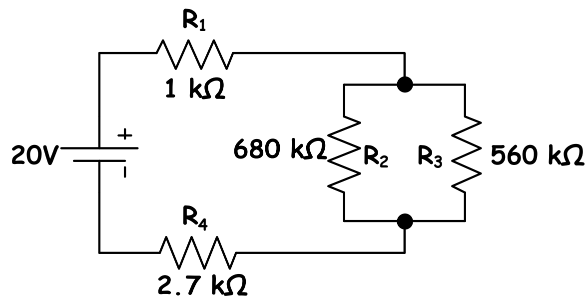
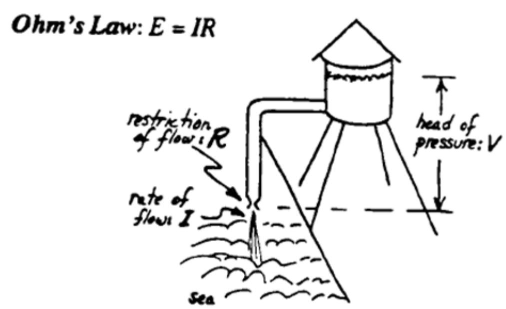
Analyzing Resistor-Only Circuits

- A typical Problem: Determine R_T, I_T, V_1, V_2, V_3, I_2, and I_3 for the circuit shown below.



- Six things you should know when analyzing Resistive Circuits.

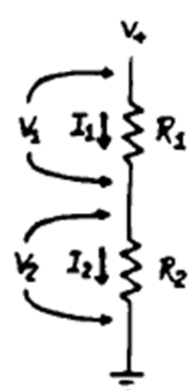
(1) Ohm's Law



$$V = IR \qquad \text{where:} \quad V = \text{Volts(v)} \quad (\text{Analagous to pressure})$$
$$I = \text{Amps(A)} \quad (\text{Analagous to rate of flow})$$
$$R = \text{Resistance}(\Omega) \quad (\text{Analagous restriction of flow})$$

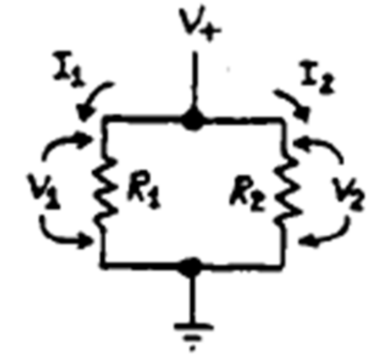
(2) Combining Resistances in Series

In a series circuit, the current is the same throughout the circuit.



$$I_{total} = I_1 = I_2$$
$$V_{total} = V_1 + V_2$$
$$R_{total} = R_1 + R_2$$

(3) Combining Resistances in Parallel



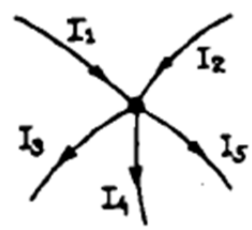
In a parallel circuit the voltage is the same across all elements.

$$I_{total} = I_1 + I_2$$
$$V_{total} = V_1 = V_2$$
$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

If more than 2 parallel resistors,

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots}$$

(4) Kirchhoff's Current Law (KCL)



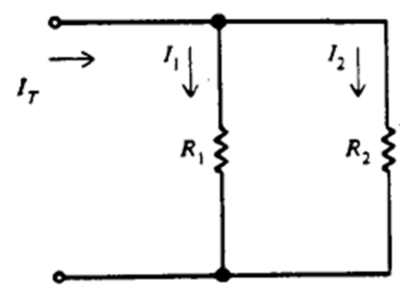
Sum of currents in & out of node is zero (algebraic sum, of course).

The algebraic sum of the currents at any node is equal to 0.

Or, another way of saying it is:

"The GOES-INT-As equals the GOES-OUT-As"

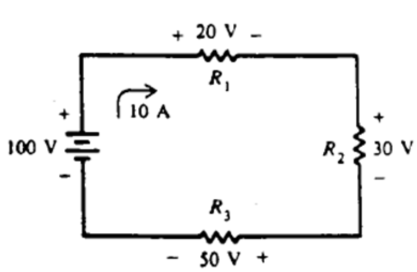
(5) Current Divider Rule (Parallel Circuits only)

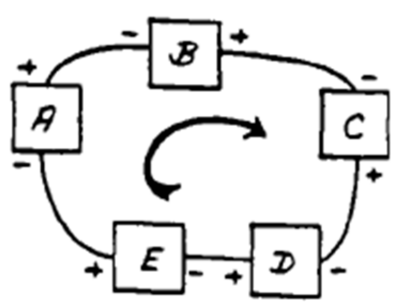


Good for only two resistors:

$$I_T = I_1 + I_2$$
$$I_{R1} = \frac{I_T \bullet R_2}{R_1 + R_2}$$

(6) Kirchhoff's Voltage Law (KVL)



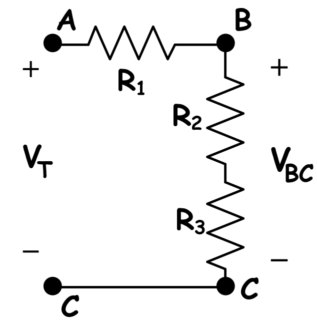


Sum of voltages around loop (circuit) is zero.

$$0 = -100 + V_{R1} + V_{R2} + V_{R3}$$
$$= -100v + 20v + 30v + 50v$$

(7) Voltage divider Rule (Series Circuits only)

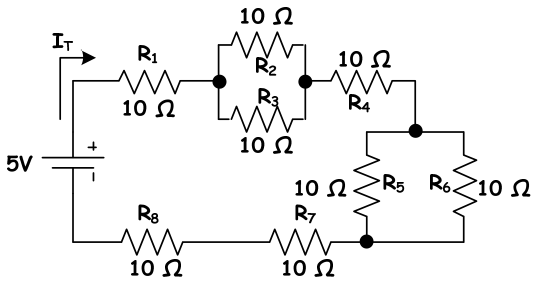
$$V_{BC} = \frac{V_T\left(R_2 + R_3\right)}{R_1 + R_2 + R_3}$$



General guidelines for solving a complex circuit

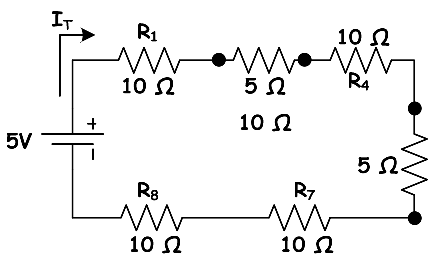
1. Find $\mathbf{R_T}$. When solving complex circuits, find $\mathbf{R_T}$ 1st. This will often involve many steps.
2. Starting as far from the source as possible, use equivalent circuits for each step of R_T. The equivalent circuits will be a great help when finding currents and voltages.
3. Find $\mathbf{I_T}$. Use the final equivalent circuit showing $\mathbf{R_T}$ to find the total current.
4. Calculate the voltage drop for any series resistors. Whatever voltage is not dropped across a series resistor will be available for the parallel combinations.
5. Using the voltage applied to the parallel combinations, determine how the current will split between the branches.
6. Use the branch currents and resistance values to determine the voltage drops for each resistor.

Problem 1: **Determine R_T, I_T, I_{R2}, I_{R7}, V_{R5}, and V_{R6} for the given circuit.**



$$R_2 \parallel R_3 = \frac{10\Omega(10\Omega)}{10\Omega + 10\Omega} = 5\Omega$$

$$R_5 \parallel R_6 = \frac{10\Omega(10\Omega)}{10\Omega + 10\Omega} = 5\Omega$$



$$\begin{aligned} R_T &= R1 + (R2 \parallel R3) + R4 + R5 \parallel R6 + R7 + R8 \\ &= 10\Omega + 5\Omega + 10\Omega + 5\Omega + 10\Omega + 10\Omega \\ &= 50\Omega \end{aligned}$$

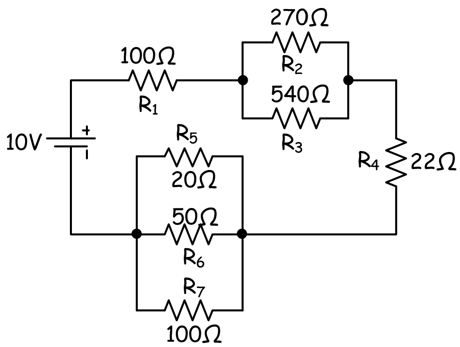
$$I_T = I_{R1} = I_{R4} = I_{R7} = I_{R8} = \frac{5v}{50\Omega} = \boxed{100mA}$$

$$I_{R2} = \frac{I_T(R3)}{R2 + R3} = \frac{100mA(10\Omega)}{10\Omega + 10\Omega} = \boxed{50mA}$$

$$V_{R5} = V_{R6} = 100mA(5\Omega) = \boxed{500mA}$$

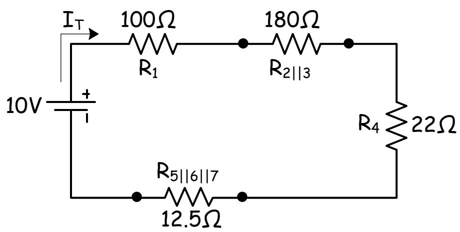
PROBLEM 2

Find R_T, I_T, I_{R3}, I_{R4}, I_{R7}, V_{R4}, and V_{R7} in the circuit shown.



$$R_2 \parallel R_3 = \frac{(270\Omega)(540\Omega)}{270\Omega + 540\Omega} = 180\Omega$$

$$R_5 \parallel R_6 \parallel R_7 = \frac{1}{\frac{1}{20\Omega} + \frac{1}{50\Omega} + \frac{1}{100\Omega}} = 12.5\Omega$$



$$R_T = R_1 + R_{2||3} + R_4 + R_{5||6||7}$$
$$= 100\Omega + 180\Omega + 22\Omega + 12.5\Omega = 314.5\Omega$$

$$I = \frac{V}{R_T} = \frac{10v}{314.5\Omega} = \boxed{31.8mA} = I_{R4}$$

$$I_{R3} = \frac{I_T R_2}{R_2 + R_3} = \frac{31.8mA(270\Omega)}{270\Omega + 540\Omega} = \boxed{10.6mA}$$

$$I_{R7} = \frac{I_T R_{5||6}}{R_{5||6} + R_7} = \frac{31.8mA(20\Omega \parallel 50\Omega)}{(20\Omega \parallel 50\Omega) + 100\Omega}$$

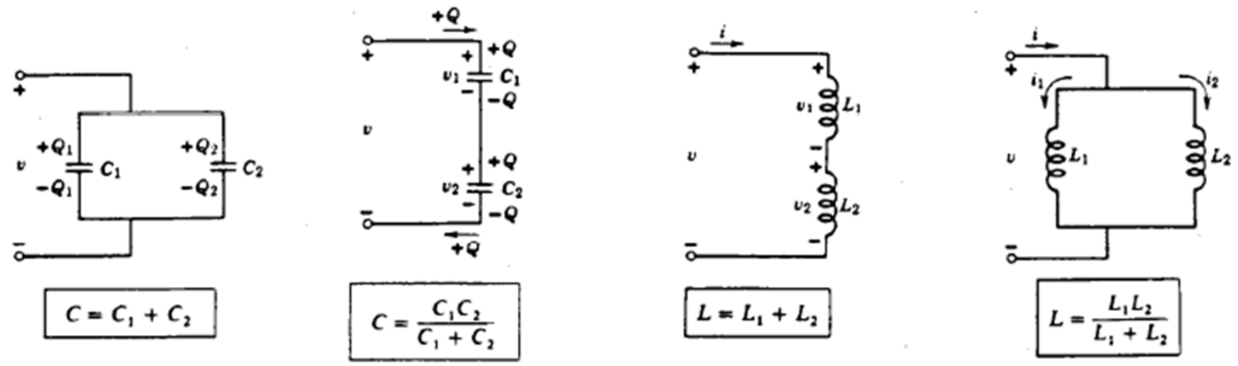
$$= \frac{31.8mA(14.29\Omega)}{14.29\Omega + 100\Omega} = \boxed{3.975mA}$$

$$VR4 = I_{R_4} R_4 = 31.8mA(22\Omega) = \boxed{699.6mV}$$

$$VR7 = I_{R_7} R_7 = 3.975mA(100\Omega) = \boxed{397.5mV}$$

Capacitors and Inductors

Combining Capacitors and Inductors in Parallel and in Series



(a) Parallel capacitors (b) Series capacitors (a) Series inductances (b) Parallel inductances

(1) Find the total capacitance of two capacitors in parallel with values of 10μF and 100μF.

$$C_T = 10\mu F + 100\mu F = \boxed{110\mu F}$$

(2) Find the total capacitance of two capacitors in series with the values of 10μF and 5000000pF.

$$C_T = \frac{1}{\frac{1}{10\mu F} + \frac{1}{5000000pF}} = \frac{1}{\frac{1}{10\mu F} + \frac{1}{5\mu F}} = \boxed{3.33\mu F}$$

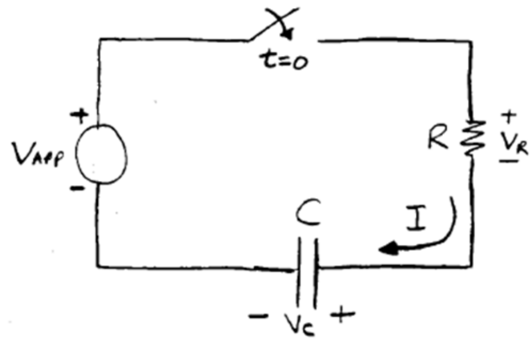
(3) Find the total inductance of two inductors in series with values of 500μH and 10mH.

$$L_T = 500\mu H + 10mH = \boxed{10.5mH}$$

CAPACITORS and INDUCTORS in D.C. CIRCUITS

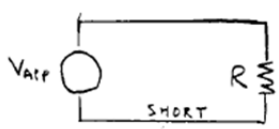
INITIAL and FINAL STATES OF AN RC CIRCUIT

The Capacitor goes from a **SHORT-CIRCUIT condition [t(0+)]** to an **OPEN-CIRCUIT condition [t(∞)]** in the face of a direct current in an RC circuit **(from switch closing to steady-state of the DC)**.

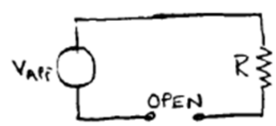


CAPACITOR	
$t=0$	$t=\infty$
$V_C = 0$	$V_C = V_{APP}$
$V_R = V_{APP}$	$V_R = 0$
$I = V_{APP}/R$	$I = 0$
Capacitor is a short circuit	*Capacitor is an open circuit*

- *at $t = 0$ circuit looks like*

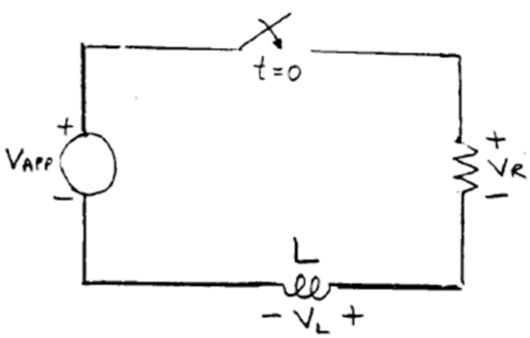


- *at $t = \infty$ circuit looks like*



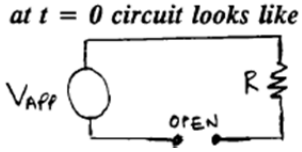
INITIAL and FINAL STATES OF AN RL CIRCUIT

The INDUCTOR goes from an **OPEN-CIRCUIT condition [t(0+)]** to a **SHORT-CIRCUIT condition [t(∞)]** in the face of a direct current in an RL circuit **(from switch closing to steady-state of the DC)**.

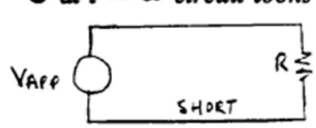


INDUCTOR	
$t=0$	$t=\infty$
$V_L = V_{APP}$	$V_L = 0$
$V_R = 0$	$V_R = V_{APP}$
$I = 0$	$I = V_{APP}/R$
Inductor is a open circuit	*Inductor is a short circuit*

- *at $t = 0$ circuit looks like*

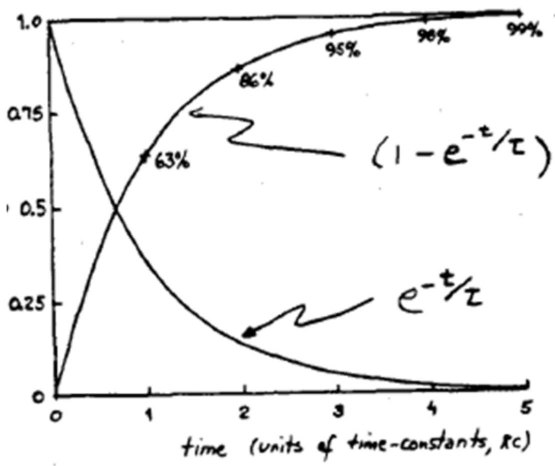


- *at $t = \infty$ circuit looks like*

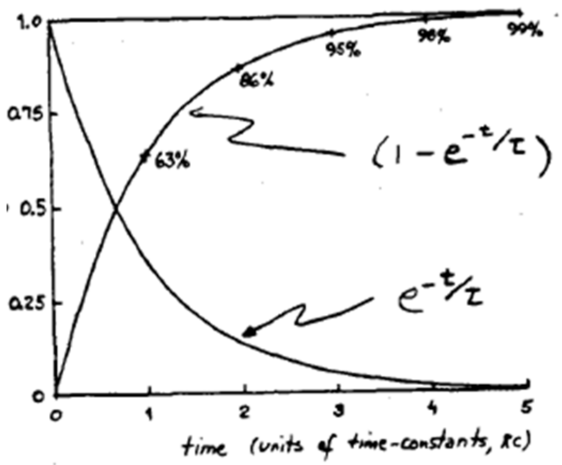


WHAT HAPPENS IN BETWEEN (t = 0+) and (t = ∞)?

RC Circuit

 	Time constant ➔ $\tau = RC$
	General equation that is useful for both current and voltage; charge and discharge. $y(t) = y(\infty) + \left[y(0^+) - y(\infty)\right] e^{-t/\tau}$
	Voltage Charge (no Initial Condition) $v_c(t) = E + \left[0 - E\right] e^{-t/RC}$
Don't try to memorize the %'s except for 2 of them: • **63% in one time constant,** • **99% in five time constants**	**Current Charge** (no Initial Condition) $i_c(t) = 0 + \left[\frac{E}{R} - 0\right] e^{-t/RC}$
Charge Cycle: **t(0+) = Short-Circuit** **t(∞) = Open-Circuit**	If there is an initial charge, it would be denoted as v(0) or i(0) and would be the initial condition [t(0+)] in the equation.

RL Circuits



Time constant ➔ $\tau = L/R$

General equation that is useful for both current and voltage; charge and discharge.

$$y(t) = y(\infty) + \left[y(0^+) - y(\infty)\right] e^{-t/\tau}$$

Voltage Charge (no Initial Condition)

$$v_L(t) = 0 + \left[E - 0\right] e^{-Rt/L}$$

Current Charge (no Initial Condition)

$$i_L(t) = \frac{E}{R} + \left[0 - \frac{E}{R}\right] e^{-t/RC}$$

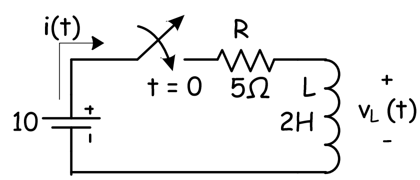
Don't try to memorize the %'s except for 2 of them:

- **63% in one time constant,**
- **99% in five time constants**

Charge Cycle:
t(0+) = Open-Circuit
t(∞) = Short-Circuit

If there is an initial charge, it would be denoted as v(0) or i(0) and would be the initial condition [t(0+)] in the equation.

Example: The expression for current in the 5 ohm resistor of the circuit below for time t > 0 is:



(A) $2e^{-2t}A$

(B) $2e^{-5t}A$

(C) $2 - 2e^{-2t}A$

(D) $2 - 2e^{-2.5t}A$

(E) $2 - \frac{5}{2}e^{-2t}A$

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{5} \Rightarrow e^{-5t/2} \Rightarrow e^{-2.5t}$$

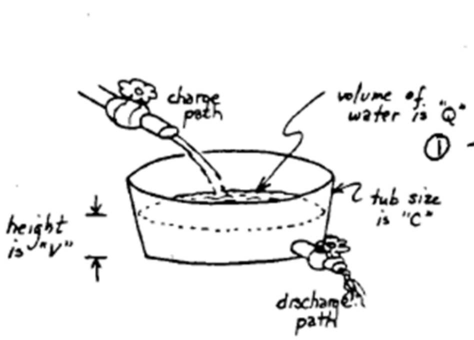
$$i(0^+) = Open - Circuit = 0A$$

$$i(\infty) = Short - Circuit = \frac{10v}{5\Omega} = 2A$$

$$i(t) = 2A + \left[0A - 2A\right] e^{-2.5t}$$

$$i(t) = 2 - 2e^{-2.5t}$$

OTHER THINGS YOU SHOULD KNOW ABOUT CAPACITORS AND INDUCTORS



The charge $q_C(t)$ and voltage $v_C(t)$ relationship for a capacitor C in farads is

(1) $$C = q_C(t)/v_C(t) \quad \text{or} \quad q_C(t) = Cv_C(t)$$

A parallel plate capacitor of area A separated a distance d by an insulator with a permittivity ϵ has a capacitance

(2) $$C = \frac{\epsilon A}{d}$$

The current-voltage relationships for a capacitor are

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(\tau)d\tau$$

and $$i_C(t) = C\,(dv_C/dt)$$

The energy stored in a capacitor is expressed in joules and

(3) $$\text{Energy} = Cv_C^2/2 = q_C^2/2C = q_Cv_C/2$$

The inductance L of a coil is

$$L = N\phi/i_L$$

and using Faraday's law, the voltage-current relations for an inductor are

$$v_L(t) = L\,(di_L/dt)$$

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau, \quad \text{where}$$

v_L = inductor voltage,
L = inductance (henries), and
i = current (amps).

The energy stored in an inductor is expressed in joules and

(4) $$\text{Energy} = Li_L^2/2$$