A VERY Short Review of Binary and Hex number systems

This review is meant only to provide enough information for non-EE or EET majors to understand enough about binary and hex number systems to be able to enter the world of truth tables, flip-flops, and Multisim's Word Generator. For this reason, it will not cover (except perhaps in passing) the OCTAL (base 8) number system or anything about fractions in number systems. For a compete coverage of these skipped topics (along with an expansion on the topics covered below) you need to refer to my set of number systems lectures provided in EET 310.

Introduction

The **total number of digits in a number system is called the radix (r) or "base" of the system.** The highest number in the system is equal to (**r** - 1). The radix of the decimal system is 10 since it has 10 digits (0 through 9).

Selected Number Systems

Name	Base (radix)	Count	
Decimal	10	0-9	
Binary	2	0-1	
Octal	8	0-7	
Hex	16	0-15	0-9, A, B, C, D, E, F

Note that $r - 1 = 10 - 1 = 9$, which is the highest number in the decimal system. What you know as the "**decimal point**" **is also known as** the "**radix point**". The radix point separates the integer portion of the number from the fraction portion. Each position in the number has a weight attached to it. This weight is equivalent to the radix of the system **raised to a power**. **This power is the position number of the digit**. It starts with a '**0**' immediately to the left of the radix point and increases by 1 as you move to the left. It decreases by one as you move to the right.

Base 10 (decimal) number system

In order to fully understand other number systems it is necessary to take a short trip into the realm of the **Base 10 (decimal) system**. Of course you have been using this system since the day you first began learning how to count on your fingers, but it never hurts to review the basics that the system was built on. Take a look at Figure 2 below. Note that each digit has a 10 to some power above it. This number is the base number of the system (10), raised to the power of the position of the digit.

$$\underbrace{3}_{10^2}\ \underbrace{4}_{10^1}\ \underbrace{2}_{10^0}$$

We can represent this number by the polynomial

$$\left(3*10^2\right)+\left(4*10^1\right)+\left(2*10^0\right)$$

Figure 2

There are many number systems other than **Decimal (Base 10)**. For the purposes of preparation for the understanding of Flip-flops and Multisim's Word Generator we will stick to two of the most common: **Binary (Base 2)** and **Hex (Base 16)**. The term "**Base**" is derived from the fact that all number systems are logarithmic in nature. For instance, the decimal number system is **log to the Base 10** ($\log_{10}x$). Binary is "**log to the Base 2**", ($\log_2 x$).

The Binary System

The **radix** of the Binary system is **2**, thus binary is known as "Base **2**" and the highest number in the system is **r - 1 = 1**. The **BInary digiT** is abbreviated as "**BIT**". Just like the decimal systems, it can be shown in positional notation.

Binary to Decimal Conversion

Conversion between the Binary number system to the Decimal number system is quite easy. Just take each bit and multiply it by two raised by the power of the position value of the bit as can be seen in the example below.

$$\underbrace{1}_{2^4}\;0\;0\;\underbrace{1}_{2^1}\;\underbrace{1}_{2^0}$$

$$\left(1 * 2^4\right) + \left(1 * 2^1\right) + \left(1 * 2^0\right) = 16 + 2 + 1 = 19_{10}$$

A couple of things to note about the above. **First, note that the radix of the system is placed as a subscript of the number. This has to be done for all systems except base 10. In fact, if more systems than base 10 are being used, you should do it for base 10 as well. If this is not done, I will assume the number to be base 10, which will make it wrong!** You have just performed a **Binary to Decimal conversion**. Didn't hurt a bit, did it? Let's do it one more time.

Convert 101101_2 to base 10

5	4	3	2	1	0	
1	0	1	1	0	1_2	
2^5		2^3	2^2		2^0	
32 +		8 +	4 +		1	$\Rightarrow$ 45_{10}

Binary to Decimal Conversion Example #2

Decimal to Binary Conversion

Most books teach a method called "**radix divide/multiply method**" to perform conversions between decimal and other number systems. I have found this method to be cumbersome and the user is very prone to making mistakes with it. In addition, it tends to take up a lot of space on a paper. The method used in this course is called the "**Sum of Weights**" method. **This is the preferred method and you are required to use this method on tests and homework in this course.** The same method can be used **ANY TIME** you desire to convert from the decimal system to some other number system.

1. **Record the list of powers of 2 up to the point of going over the given decimal value**
2. **Divide that number into the Decimal number. It will go in "1" time on the 1st division. If it goes in more than "1" time, you did something wrong.**
3. **Record the remainder. This will be your new number to divide into. This time it will either go in "1" time or "0" times. Repeat until you finish the number.**

Convert 182_{10} to Binary (Base 2)

remainder	⇒	182	54	54	22	6	6	2	0		
powers of two	⇒	128	64	32	16	8	4	2	1	=	$\boxed{182_{10}}$
		1	0	1	1	0	1	1	0_2		

Decimal to Binary Conversion Example #1

Convert 42_{10} to Binary (Base 2)

remainder	⇒	42	10	10	2	2	0		
powers of two	⇒	32	16	8	4	2	1	=	42_{10}
		1	0	1	0	1	0		

Decimal to Binary Conversion Example #2

Very often it is very important to keep track of the number of bits. A very simple way of performing the conversion is shown in Example 3.

bit	8	7	6	5	4	3	2	1	0
#	273	17	17	17	17	1	1	1	1
power	256	128	64	32	16	8	4	2	1
273	1	0	0	0	1	0	0	0	1_2

Decimal to Binary Conversion Example #3

The Hexadecimal (HEX) System

Now lets look at a system that is becoming more and more important, the Base 16, or Hex system. At first glance you might think that Hex is the same as decimal. However, there are two differences. The first is that the largest number represented in Hex is **r - 1 = 16 - 1 = 15.** Since decimal numbers only go to 9, we need to find a way to represent 10 thru 15. We do this with capital letters **A(10) thru F(15)**. The second difference is that conversion between Hex and Binary is **SIGNIFICANTLY easier**. **Hex to decimal on the other hand is still comparatively difficult.**

Hex to Decimal conversion

$$\underbrace{A}_{16^2}\underbrace{C}_{16^1}\underbrace{3}_{16^0}$$

$$\left(A * 16^2\right) + \left(C * 16^1\right) + \left(3 * 16^0\right)$$

$$\left(10 * 256\right) + \left(12 * 16\right) + \left(3 * 1\right)$$

$$2560 + 192 + 3 = 2755_{10}$$

Hex to Decimal Conversion Example

Decimal to Hex conversion

Convert 2748 to Hex.

As with binary, use the "Sum of Weights" method.

$$\frac{2748}{256} \quad \frac{188}{16} \quad \frac{12}{1} = ABC_{16} \Leftarrow$$

$$A \quad B \quad C$$

Decimal to Hex conversion example

Binary to Hex conversion

In order to convert from Binary to Hex you need to consider that the largest Hex digit is a 15. It takes four binary bits to represent a 15 so you start at the radix point and group the bits into groups of 4.

Convert 101101_2 to Hex

$$\underbrace{10}_{2}\,\underbrace{1101}_{D} \Rightarrow 2D_{16}$$

Hex to Binary conversion

Again, we keep in mind the groups of four except this time in reverse.

Convert $5AC_{16}$ $\Rightarrow$ Base 2

$$\underbrace{5}_{0101}\quad\underbrace{A}_{1010}\quad\underbrace{C}_{1100} \Rightarrow 10110101100_2$$

To finish up, lets look at the count sequence from 0 to 15 :

Base 16	Base 10	A	B	C	D	A	B	C	D	Base 10	Base 16
0	0	0	0	0	0	1	0	0	0	8	8
1	1	0	0	0	1	1	0	0	1	9	9
2	2	0	0	1	0	1	0	1	0	10	A
3	3	0	0	1	1	1	0	1	1	11	b
4	4	0	1	0	0	1	1	0	0	12	C
5	5	0	1	0	1	1	1	0	1	13	d
6	6	0	1	1	0	1	1	1	0	14	E
7	7	0	1	1	1	1	1	1	1	15	F